Impact of Higher Order Beliefs
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Outline

- "Global Games" and Higher Order Beliefs
- Bounded Rationality
- Taking Harsanyi Seriously
- Strategic versus Structural Uncertainty
- Experiments

Common Knowledge of Rationality and Payoffs

	Invest	Not Invest
Invest	θ, θ	$\theta - 1, 0$
Not Invest	$0, \theta - 1$	$0, 0$

- Common Knowledge that $\theta > 1$, "invest" is dominant strategy
- Common Knowledge that $\theta < 0$, "not invest" is dominant strategy
- Common Knowledge that $\theta \in [0, 1]$, both actions are fully rational

Common Knowledge of Rationality, Not Payoffs

- A Harsanyi-Mertens-Zamir type describes beliefs about θ, beliefs about θ and opponent's beliefs about θ,...

- Necessary conditions for "Invest" to be rationalizable for i:

  1. $E_i(\theta) \geq 0$.
  2. $E_i(\theta) \geq 1 - \mathrm{Pr}_i(E_j(\theta) \geq 0)$
  3. $E_i(\theta) \geq 1 - \mathrm{Pr}_i(E_j(\theta) \geq 1 - \mathrm{Pr}_j(E_i(\theta) \geq 0))$
  4. ....

- $\widehat{B}_I(A) = \{t_i : E_i(\theta|t_i) \geq 1 - \mathrm{Pr}_i(t_j \in A|t_i)\}$

- Invest is rationalizable at $\widehat{B}_I^{\infty}(\Omega)$

- $\widehat{B}_N(A) = \{t_i : E_i(\theta|t_i) \leq \Pr_i(t_j \in A|t_i)\}$

- Not Invest is rationalizable at $\widehat{B}_N^{\infty}(\Omega)$

Asymmetric Information I

- $\theta \sim U(\mathbb{R})$
- $x_i = \theta + \varepsilon_i$
- $\varepsilon_i \sim N\left(0, \frac{1}{\beta}\right)$
- Unique equilibrium
- Natural comparative statics in more complicated coordination games
- As in electronic mail game, no event is common p-belief for $p > \frac{1}{2}$:

– Individual observing x_1 thinks $\theta \sim N\left(x_1, \frac{1}{\beta}\right)$

– Prob $(\theta \leq \theta^* | x_1) = \Phi\left(\sqrt{\beta}\left(\theta^* - x_1\right)\right)$

–

$$[B_1^p]\left(\{(\theta, x_1, x_2) : \theta \leq \theta^*\}\right) = \left\{(\theta, x_1, x_2) : x_1 \leq \theta^* - \frac{1}{\sqrt{\beta}}\Phi^{-1}(p)\right\}$$

– Individual observing x_1 thinks $x_2 \sim N\left(x_1, \frac{2}{\beta}\right)$

– Prob $(\theta \leq \theta^* | x_1) = \Phi\left(\sqrt{\frac{\beta}{2}}\left(\theta^* - x_1\right)\right)$

–

$$B_1^p\left(\{(\theta, x_1, x_2) : x_2 \leq x^*\}\right) = \left\{(\theta, x_1, x_2) : x_1 \leq x^* - \frac{\sqrt{2}}{\sqrt{\beta}}\Phi^{-1}(p)\right\}$$

$$\begin{aligned} & [B_*^p]^k\left(\{(\theta, x_1, x_2) : \theta \leq \theta^*\}\right) \\ = \quad & \left\{(\theta, x_1, x_2) : x_i \leq \theta^* - \frac{1 + (k-1)\sqrt{2}}{\sqrt{\beta}}\Phi^{-1}(p)\right\} \end{aligned}$$

Asymmetric Information II

- $\theta \sim N\left(y, \frac{1}{\alpha}\right)$
- $x_i = \theta + \varepsilon_i$
- $\varepsilon_i \sim N\left(0, \frac{1}{\beta}\right)$
- Uniqueness if

$$\frac{\alpha^2}{\beta}\left(\frac{\alpha+\beta}{\alpha+2\beta}\right) \leq 2\pi$$

Alternate Uniqueness Conditions

- Let

$$\begin{aligned}\overline{\theta}_i(t_i) &= E_i(\theta|t_i) \\ p_i(t_i) &= \Pr_i\left\{\overline{\theta}_j(t_j) \geq \overline{\theta}_i(t_i)\,|t_i\right\}\end{aligned}$$

- Common Knowledge that $p_1 = p_2 = \frac{1}{2} \Rightarrow$ type t_i invests if and only if $\overline{\theta}_i(t_i) > \frac{1}{2}$.

- Common Knowledge that $\alpha \leq p_i \leq 1 - \alpha \Rightarrow$ type t_i invests if $\overline{\theta}_i(t_i) > 1 - \alpha$ and does not invest if $\overline{\theta}_i(t_i) < \alpha$

Modelling

- Leamer (1985): "it is indeed a frightful sight to observe economists tiptoeing into the edges of the quagmire of philosophy"
- Rubinstein (1991): models as players' perception of reality
- Two approaches:
  1. Bounded Rationality
  2. Full Rationality:
     - Model endogeneity of asymmetric information.
     - What does the universal type space really look like?

Bounded Rationality

- "As if common knowledge"
- "Common knowledge if high number of levels of knowledge": Rubinstein (1989)
- "Confidence" take risky but high expected value action only if there is high "confidence" that returns are high
  - confidence = common p-belief?
- Act as if common knowledge that $p_1 = p_2 = \frac{1}{2}$
  - Jehiel and Koessler (2004)... "analogy-based expectation equilibrium"

  - Reducing dimension of tyspe favors uniqueness results

- "Strategic uncertainty:" act as if exogenous uncertainty about others' behavior

Full Rationality

- What do types in the universal type space look like? Are funny looking type spaces (the email game, Carlsson-van Damme) more representative of the universal type space than "nice" type spaces?

- Morris (2002), Dekel, Fudenberg and Morris (2005)

  1. "Strategic" metric topology: two types are close if they have same ε-rationalizable actions in all games
  2. "Finite" types are dense in strategic topology, but not category 1

Strategic versus Structural Uncertainty

- In our global game, behave as if 50/50 probability distribution over opponent's action

- More generally, "Laplacian" beliefs over opponents' actions in global game

  - $I = [0,1]$, $\theta \sim g(\cdot)$, $x_i = \theta + \sigma\varepsilon_i$, $E_i\left(\#\left\{j : x_j \leq c\right\}|x_i|\right) \approx c$ for small σ

- are global games just about introducing strategic uncertainty?

- general modelling question: repeated games, reputation, etc...

- Is there a meaningful distinction between strategic and structural uncertainty?
  - complete information
  - purification
  - with rich higher order beliefs, distinction goes away?
    * intuition: can always use tails of higher order beliefs to proxy for "strategic uncertainty"
    * formalization: Weinstein/Yildiz show you do not need to add payoff-irrelevant types to support rationalizable play on universal type space.

Experiments

- Beauty contest experiments and levels of beliefs
- Coordination Games
  - With complete information, behave as if strategic uncertainty
  - Heinemann, Nagel and Ockenfels (2004)
- Measuring "Publicness"
  - Chaudhuri, Schotter and Sopher (2001)

Conclusions

- Higher order beliefs seems to matter
- Applied economists are adept at finding tools to tame higher order beliefs (in the name or tractability)
- Useful to develop tractable models where higher order beliefs matter
- Must think about how to interpret those models.