Auctions with Multiple Objects

Nemmers Prize Conference
in honor of Paul Milgrom

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University of Maryland
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Introduction

- A wave of theoretical research into auctions had concluded in the 1980's, by which time there was a widespread sense that it had become a relatively complete body of work with very little remaining to be discovered
- … but two pivotal events intervened at the start of the 1990's, changing this perception:
  - the Salomon Brothers scandal in the US Government securities market in 1991; and
  - the advent of the Federal Communications Commission (FCC) spectrum auctions in 1994

Introduction

Salomon Brothers scandal (1991)

- US Treasury auctions were conducted as sealed-bid, pay-as-bid auctions, with each bidder limited to bidding for 35% of supply
- On some instances, Salomon Brothers had placed bids for as much as 105% of supply, with the intent of “cornering” the market
- In the aftermath, the US Treasury and the Fed sought to change the procedures, with the input of academics

Introduction

Advent of FCC auctions

- Congress passed a bill in 1993, authorizing the FCC to allocate spectrum licenses via auction (instead of using beauty contests or lotteries)
- Spectrum licenses cover assorted geographic areas, and there are typically multiple licenses for a given geographic area
- In the preparation for auctions in 1994, the FCC (and telecom bidders) sought input as to a procedure for selling these licenses

Introduction

- The advice of academics contributed to good outcomes
- In the case of the FCC auctions, it resulted in what is widely regarded as one of the unambiguous success stories of economics and game theory
- In the case of Treasury auctions, it contributed to the initiation of experimentation with and eventual adoption of uniform-price auctions

Introduction

- **At the same time, these two pivotal events underscored some extremely serious limitations in auction theory as it existed in the early 1990's. It became apparent then that the theory that had been developed was almost exclusively one of *single-item auctions*, and that relatively little had been established concerning *multi-unit or multi-item auctions***
- **As such, these events marked the beginning of major progress on understanding multiple-object auctions**

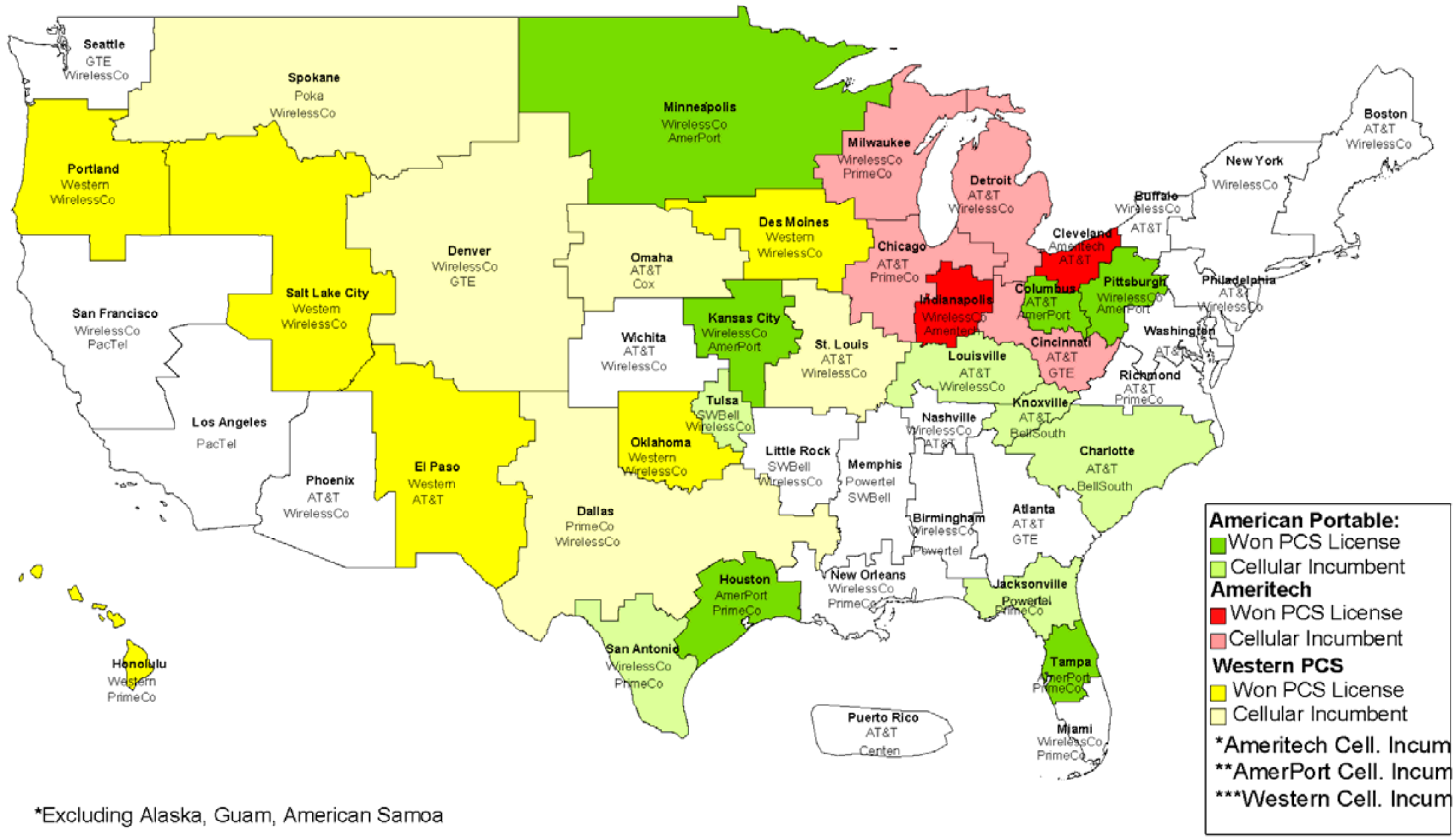
Introduction

- Given the honoree of today’s conference, my talk today will focus on what could be called the “market-design-oriented” literature on auctions for multiple objects, in particular:
  - The simultaneous ascending auction
  - Multi-unit auctions
  - Clock auctions
  - Package bidding
  - Open issues / directions

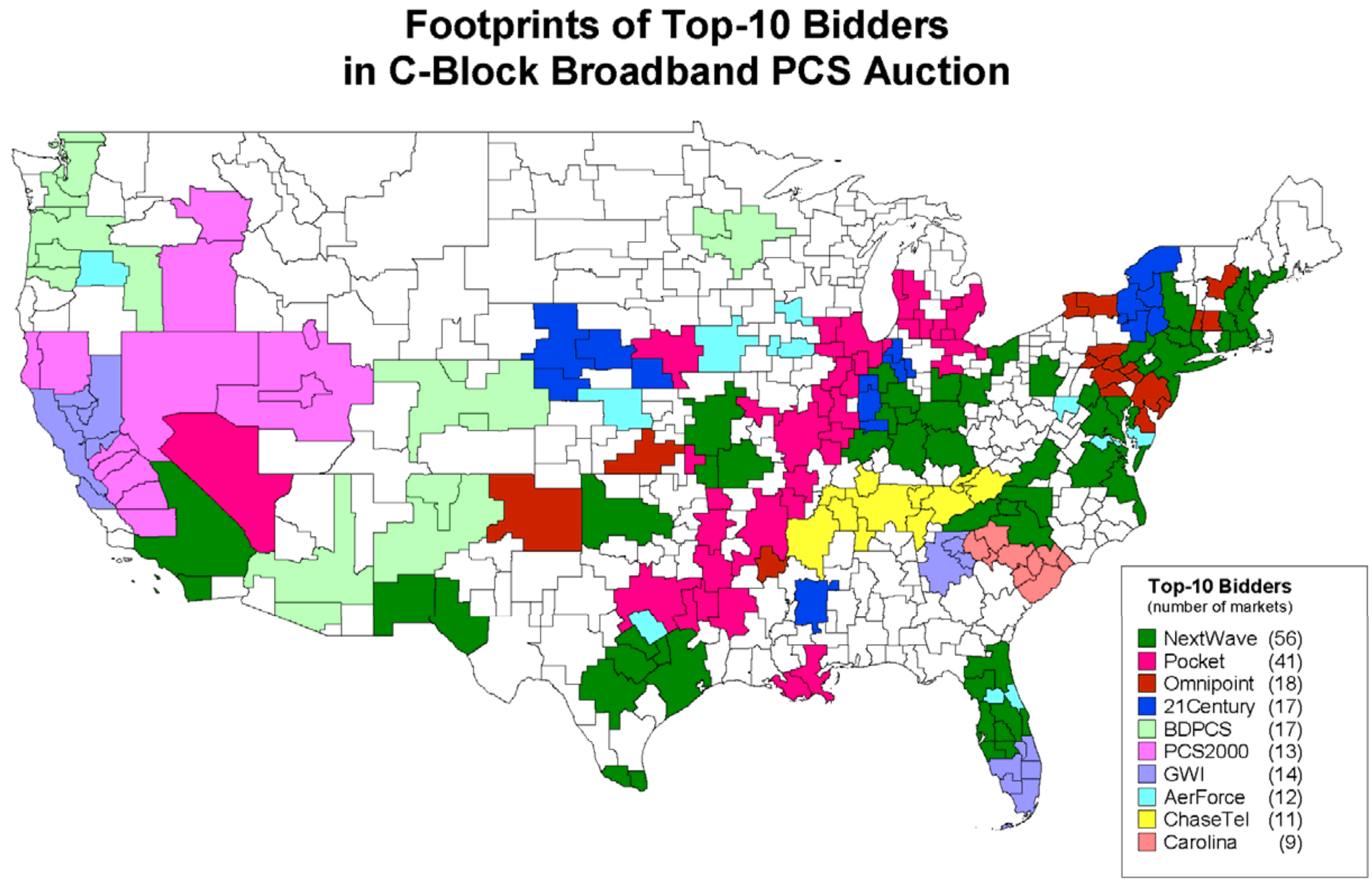
The Simultaneous Ascending Auction

A/B-Block Auction (two licenses per region)

Winning Bidders and Sample Footprints in MTA Broadband PCS Auction*



C-Block Auction (one license per region)



The Simultaneous Ascending Auction

- Description of the Simultaneous Ascending Auction (credited to Milgrom, Wilson, McAfee and McMillan)
  - All licenses are auctioned *simultaneously*
  - In each round, any bidder can raise the high bid on any license (subject to eligibility and activity rules)
  - Bidders have an eligibility based on their deposit
  - Bidders must keep active to maintain their eligibility:

    Activity = Standing High Bids + New Bids
  - Bid withdrawal penalties
  - Minimum bid increments specified for each license
  - Stopping Rule: Auction does not end on *any* license until bidding stops on *all* licenses

The Simultaneous Ascending Auction

- The “activity rule” is regarded to be the key feature:
  - Each license is assigned a number of points
  - Activity = Standing High Bids + New Bids (expressed in points)
  - Activity in a given round must be at least x% of the bidder’s eligibility (x is generally 80% early in the auction and 95% later in the auction)
  - A bidder whose activity is less than that required has its eligibility permanently reduced, commensurately
  - In short, in order for a bidder to be able to bid on licenses late in the auction, the bidder is required to bid early in the auction

Results (with discrete goods)

Theorem: Suppose that for every bidder the goods are substitutes. Then there exists a Walrasian equilibrium (Kelso and Crawford 1982, Gul and Stacchetti 1999, Milgrom 2000).

Theorem: Conversely, suppose that the set of possible valuation functions of bidders includes all substitutes preferences and at least one other valuation function. Then, if there are at least three bidders, there exists a profile of valuations such that no Walrasian equilibrium exists (Milgrom 2000).

Results (with discrete goods)

Straightforward bidding **means that, in every round of the SAA, the bidder places new bids (at the minimum price) on each element of its demand set for which it is not already the standing high bidder**

Theorem: Straightforward bidding is feasible after all histories of the SAA if and only if the goods are substitutes (Milgrom 2000; generalized by Hatfield and Milgrom 2005).

Theorem: If bidders have substitute preferences and bid straightforwardly, then the SAA terminates at a Walrasian equilibrium (as adjusted for the bid increment) and efficiency is achieved (Milgrom 2000).

The Simultaneous Ascending Auction

- **Got it right (in several critical respects):**
  - **Established and implemented the principle of offering all the items together (items are auctioned simultaneously, not sequentially)**
  - **Put a deserved emphasis on “activity rules” (anticipated the problems of “bid-sniping”, which make a mockery of dynamic auctions, two years before the advent of eBay)**
  - **Outcomes could probably be improved by package bidding, but demonstrably superior package bidding designs were not ready**

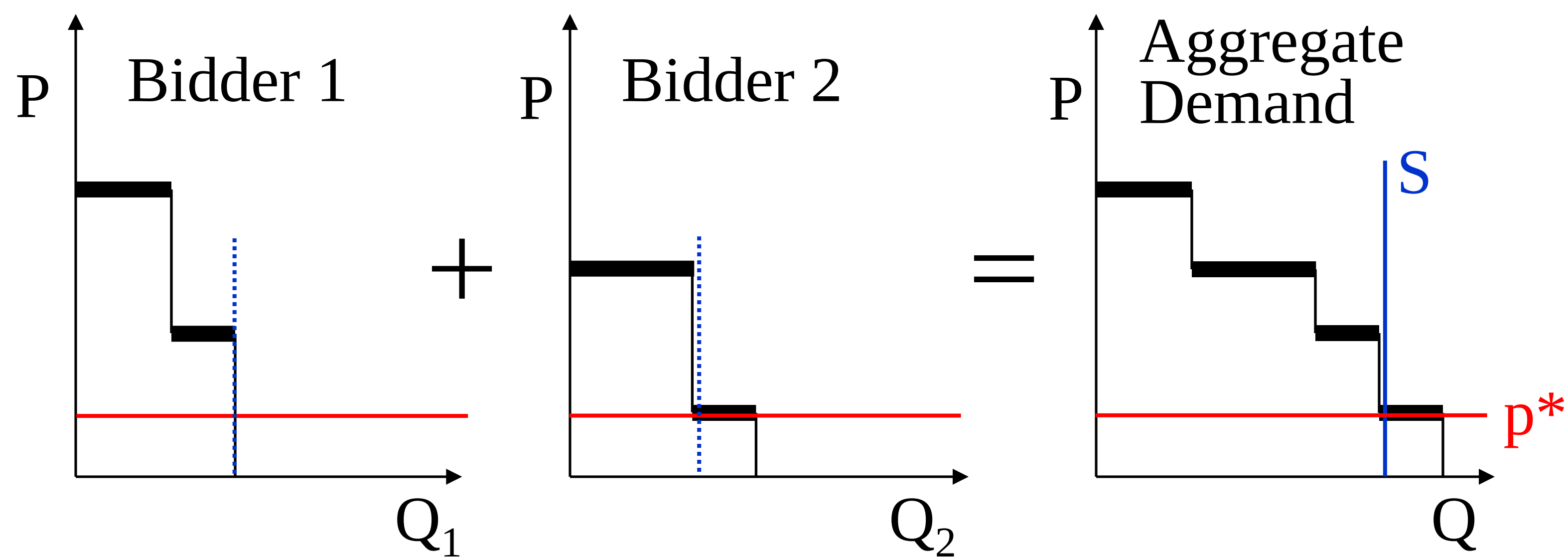
The Simultaneous Ascending Auction

- Very positive legacy:
  - The FCC auction experience has been put forward as one of the ‘success stories’ of NSF support for economic research, etc.
  - A lot more items are auctioned today than in the past, and in a significant number (but still minority) of cases, market designs are selected which reflect sophisticated modern thought

Multi-Unit Auctions

Multi-Unit Auctions

- Sealed-bid: bidders submit demand schedules
  - Pay-as-bid auction (traditional Treasury practice)
  - Uniform-price auction (Treasury in recent years)
  - Vickrey auction (William Vickrey 1961)

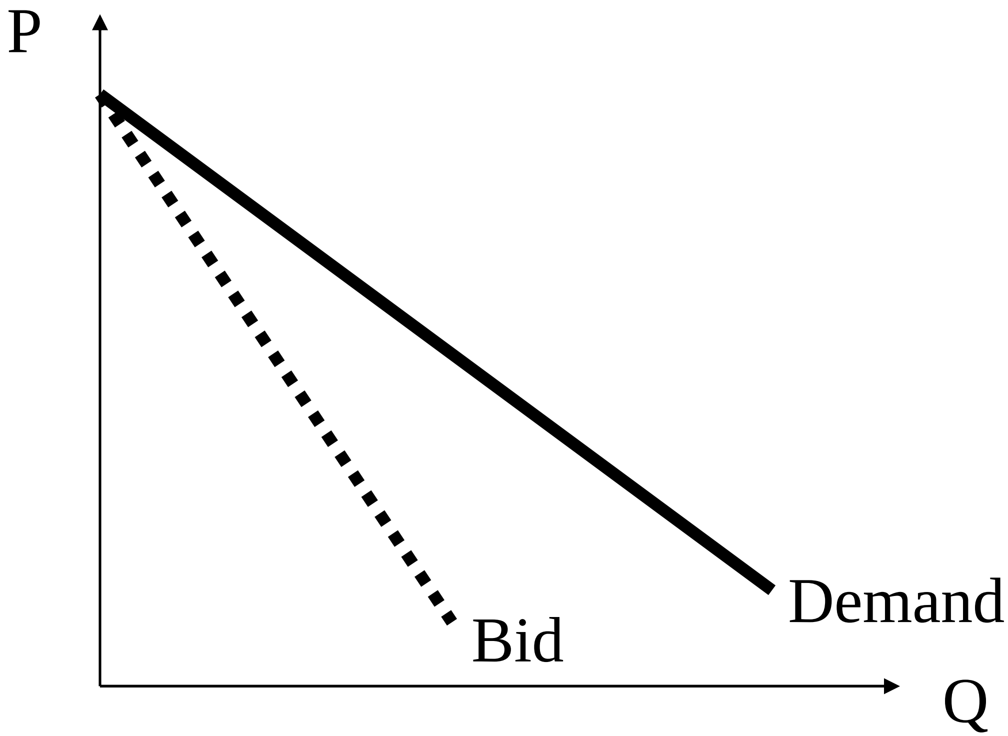


Multi-Unit Auctions

- Almost all serious discussion at the time of the Salomon Brothers scandal was argued by analogy from single-item auctions:
  - Uniform-price was ‘like’ a 2nd-price auction
  - Therefore, “you just bid what you think it’s worth”
  - Pay-as-bid was ‘like’ a 1st-price auction
  - Advantages of each was alleged to parallel the relative advantage of the 2nd-price and 1st-price auctions
  - For example, uniform-price auction was alleged to lead to efficiency

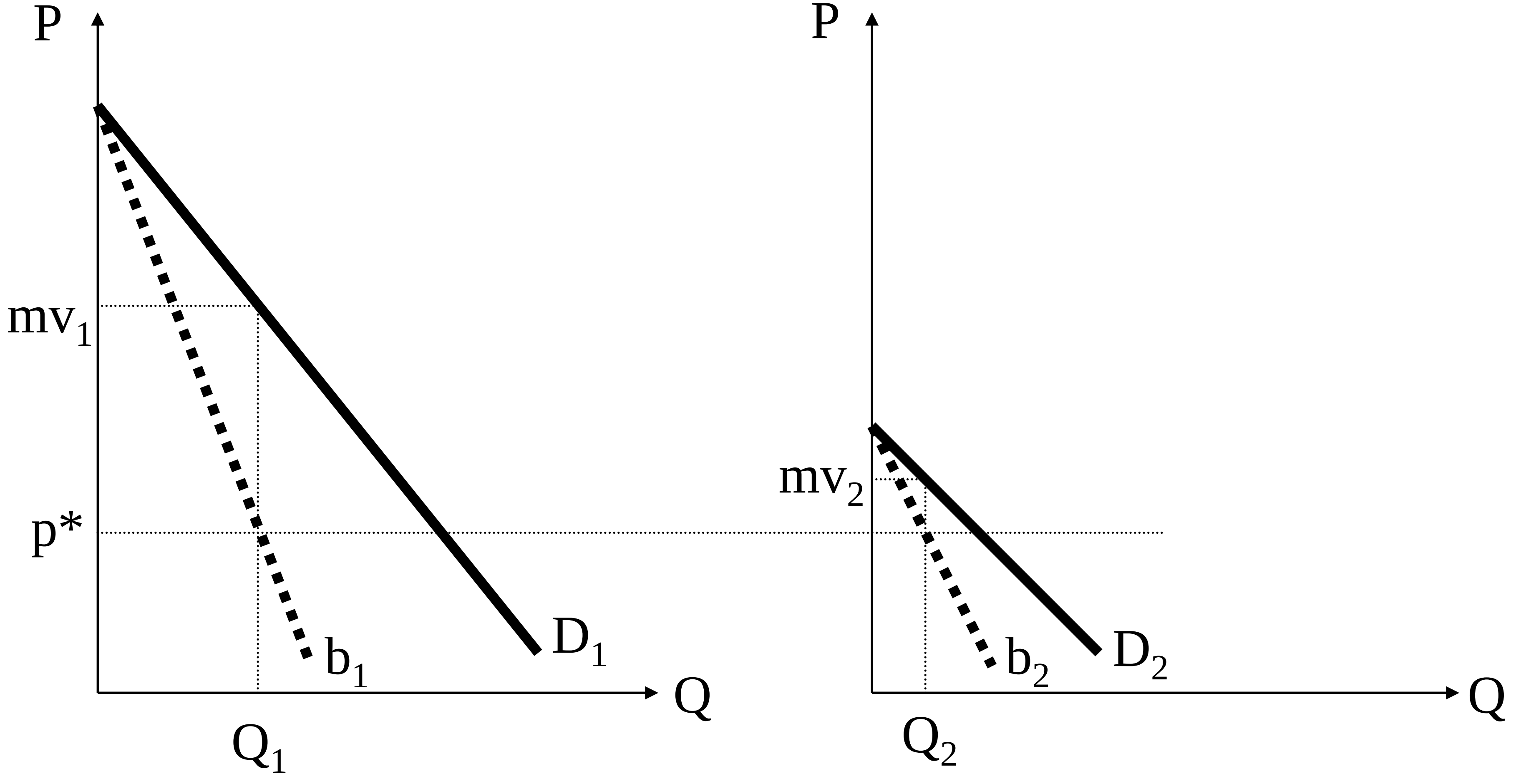
Demand Reduction in Uniform-Price Auctions

- **Qualitative nature of optimal bidding strategy in a uniform-price auction:**



Inefficiency from Differential Bid Shading

- **High-value bidder makes room for low-value rival:**

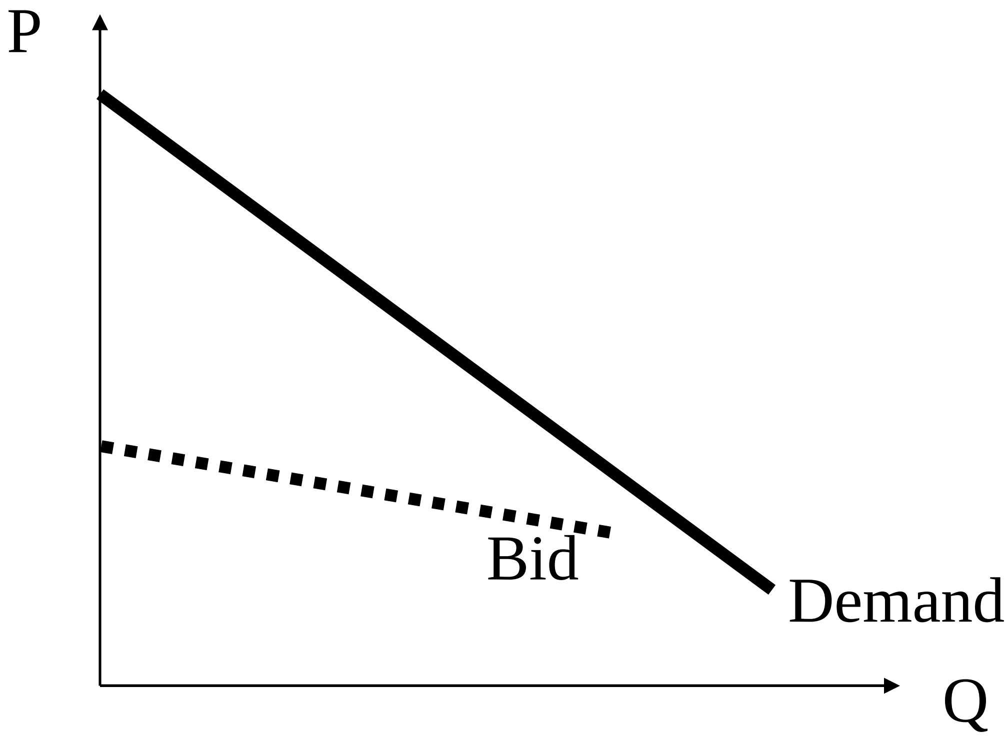


Inefficiency from Demand Reduction

- **Theorem: In any equilibrium of the uniform-price auction, with positive probability objects are won by bidders other than those with highest values (Ausubel and Cramton, 1996)**
  - **Winning bidder influences price with positive prob.**
  - **Creates incentive to shade bid**
  - **Incentive to shade increases with additional units**
  - **Differential shading implies inefficiency**
- **Exceptions to inefficiency:**
  - **Pure common value**
  - **Bidders demand only a single unit**

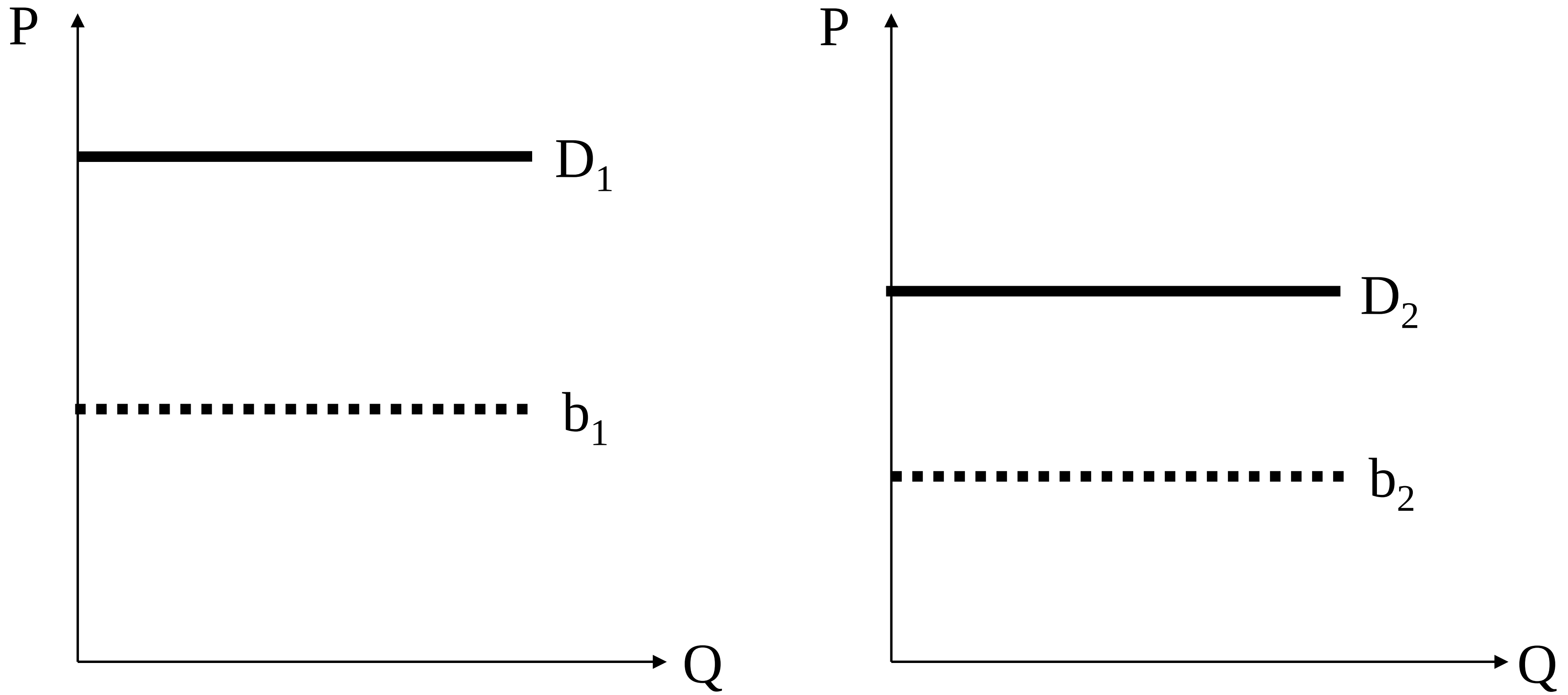
Pay-as-Bid Auction

- **Qualitative nature of optimal bidding strategy in a pay-as-bid auction:**



Pay-as-Bid Auction

- **Does not necessarily give rise to inefficiency, as bids may be ranked in same way as values:**



Implications

- There is no clear ranking of uniform-price vs. pay-as-bid auctions (it depends on environment and distributions)
- Advantages of a given format may depend on other factors (e.g., incentives for info acquisition, forward contracting)
- One should not dismiss the multi-unit Vickrey auction as an auction format
- Points to that the relationship between the simultaneous ascending auction and Walrasian equilibria may not be entirely helpful — if you run an SAA auction for multiple units, extreme demand reduction may occur

Empirical Example of Extreme Demand Reduction

- October 1999 German simultaneous ascending auction of capacity to the four GSM incumbents:

10 licenses:	nine 2×1 MHz
(almost identical)	one 2×1.4 MHz
2 high-value bidders:	Mannesmann
	T-Mobil

(See Jehiel and Moldovanu)

Empirical Example of Extreme Demand Reduction

	Licenses		
	1 2 3 4 5	6 7 8 9	10
Round 1 Mannesmann	36,360,000	40,000,000	56,000,000

Empirical Example of Extreme Demand Reduction

	Licenses 1 2 3 4 5	6 7 8 9	10
Round 2 T-Mobil	40,010,000	40,000,000	56,000,000
Round 1 Mannesmann	36,360,000		

Empirical Example of Extreme Demand Reduction

	Licenses 1 2 3 4 5 6 7 8 9 10		
Round 3			
Round 2 T-Mobil	40,010,000	40,000,000	56,000,000
Round 1 Mannesmann	36,360,000		

Recent Work

- Empirical work on uniform-price vs. pay-as-bid auctions has been divided:
  - Compare Goldreich (2007, US government securities) with Hortascu (2002, Turkish government securities)
  - The US Treasury “experiment” is unpersuasive (Malvey and Archibald, 1998)

Recent Work

- Some recent theoretical work favors pay-as-bid auctions:
  - Holmberg (2008) obtains uniqueness in uniform-price auctions, exploiting the boundary condition provided by price caps (an appropriate assumption in day-ahead electricity markets). Holmberg (2009) solves for equilibria of pay-as-bid auctions in the same environment
  - Rostek, Weretka and Pycia (2009) obtain a characterization of linear equilibria in both pay-as-bid and uniform-price auctions
  - But both lines of work take the Klemperer-Meyer (1989) “supply function equilibrium” approach of assuming common uncertainty about demand (supply) and assuming no private information. Perhaps a reasonable assumption for day-ahead electricity mkts, but somewhat tenuous for Treasury auctions and other securities markets

Clock Auctions

Clock Auctions

- In a clock auction, the auctioneer announces prices; and bidders respond with quantities
  - One or more types of items
  - In each round, auctioneer announces a price vector
  - Bidders respond by submitting quantity vectors
  - Auctioneer adjusts price vector according to excess demand
  - Process repeated until market approximately clears
- Two clear differences from SAA:
  - Auctioneer names prices, not the bidders
  - With multiple similar items, bidders bid quantities (very helpful for energy, financial products)

Clock Auctions

- Another basic difference of clock auction:
  - Nobody is taken to be the high bidder after a round; rather, *every* bidder needs to continue bidding
- Advantages:
  - Needs fewer rounds, by avoiding cycling among bidders
  - Since there is no "high bidder" for individual items, bids can be taken as package bids (no exposure problem)
  - Simpler and richer activity rules are possible
  - Easy to integrate other (e.g. efficient) payment rules
- Disadvantage:
  - Since there is no high bidder held to bid, there may be "undersell" of an item which met the reserve price

Clock Auctions

Convergence to equilibrium under straightforward bidding:

- **Theorem: If bidders have substitute preferences and bid straightforwardly, then a continuous ascending clock auction terminates at a Walrasian equilibrium and efficiency is achieved (Arrow, Block and Hurwicz 1959 for divisible goods; Gul and Stacchetti 2000 for discrete goods)**
- **However, observe that in uniform-price clock auctions, straightforward bidding should *not* be expected**

Efficient Clock Auctions

Suppose 2 units available

Price	Bidder 1	Bidder 2	Bidder 3
100	2	1	1

Efficient Clock Auctions

Suppose 2 units available

Price	Bidder 1	Bidder 2	Bidder 3
100	2	1	1
110	2 Bidder 1 “clinches” a unit	1	—

Efficient Clock Auctions

Suppose 2 units available

Price	Bidder 1	Bidder 2	Bidder 3
100	2	1	1
110	2 Bidder 1 "clinches" a unit	1	—
150	2 Bidder 1 "clinches" a second unit	—	—

Efficient Clock Auctions

Results:

- **With diminishing marginal valuations, straightforward bidding is an equilibrium of the game, yielding full efficiency**
- **With a particular formulation of the game, straightforward bidding is the unique outcome of iterated elimination of weakly dominated strategies (Ausubel 2004)**
- **A similar, but somewhat less clean, result holds for multiple types of objects when bidders have substitute preferences (Ausubel 2006)**

Package Bidding

Origins of Package Bidding

(1) VCG Mechanism (Vickrey 1961, Clarke 1971, Groves 1973)

- **Each bidder i submits bids $b_i(x)$ on all bundles**
- **Auctioneer chooses the feasible allocation $x^* \in X$ that maximizes the total bid accepted**
- **Payments are selected so that each bidder receives the incremental surplus that the bidder creates by participating in the mechanism**

- **<u>Theorem</u>: This is a dominant-strategy mechanism**

Origins of Package Bidding

(2) Experimental-based literature in the 1980s

- Rassenti, Smith and Bulfin (1982) (and Stephen Rassenti's dissertation): First article exploring package bidding as a practical auction design — for allocating airport takeoff and landing slots — demonstrating that a package design could perform better than individual slot sales
- Banks, Ledyard and Porter (1989): Explored two iterative package auction designs. The ascending package auctions outperformed the alternatives in realizing efficiency gains

Package Bidding

- One of the driving forces behind interest in package bidding has been spectrum auctions
  - Spectrum auctions are emblematic of environments with strong, varying complementarities among items
  - A package bid is an all-or-nothing bid for a set of items
  - Thus, package bidding provides the hope of defeating the “free-rider” (threshold) problem and the “exposure” problem present in single-item bidding
  - At the same time, package bidding can help to alleviate the demand reduction problem

As Paul said yesterday …

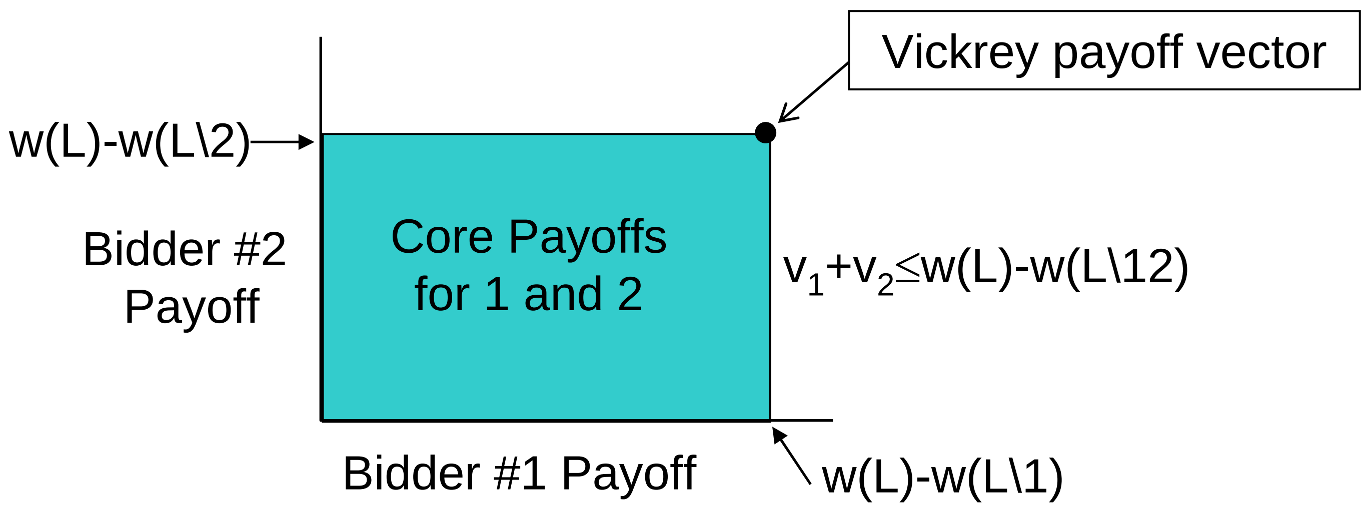
- The Vickrey auction may suffer from low revenues and extreme forms of manipulation (loser collusion and shill bidding)

The Substitutes Condition

- In understanding the outcomes of package bidding, environments divide into two cases:
  - Substitutes: The price-theory notion of substitutes, where each item is treated as a unique good, as discussed above
  - Non-Substitutes: *Everything else*
- Characterization: Goods are substitutes for bidder *i* if and only if the corresponding indirect utility function, $v_i(p)$, is submodular (Ausubel and Milgrom, 2002)

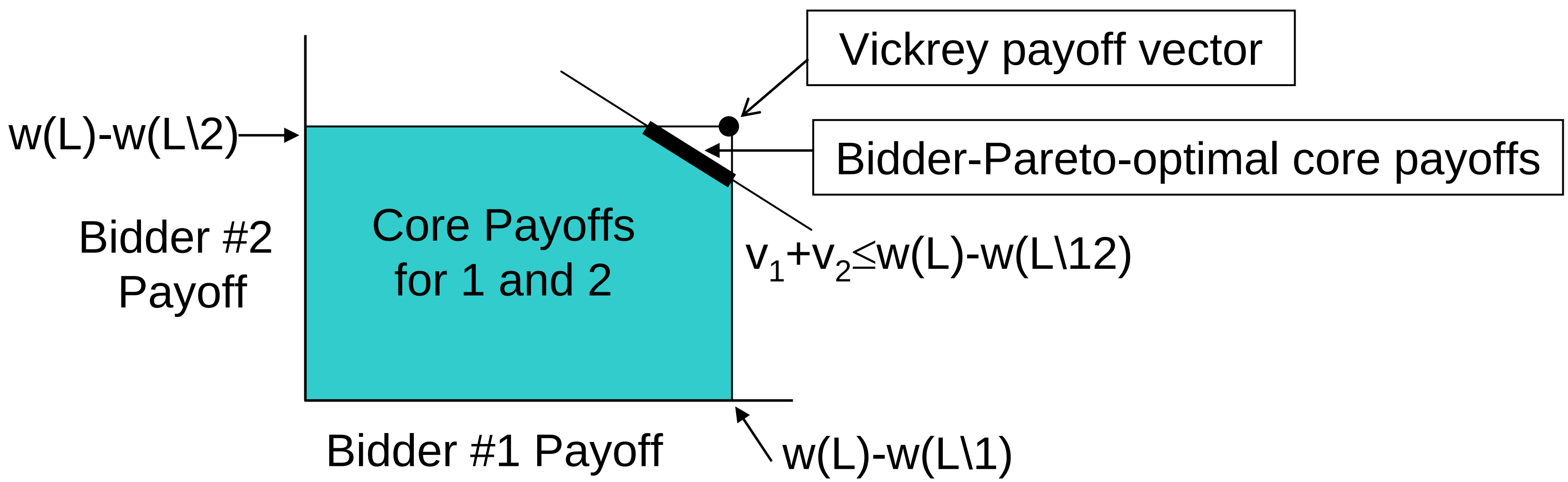
Case of Substitutes

- **If goods are substitutes, then the Vickrey payoff profile is the bidder-Pareto-optimal point in core**



Case of Non-Substitutes

- **If goods are not substitutes, then the Vickrey payoff profile is not in the core**



Ascending Proxy Auctions

- **Each bidder reports its values (and relevant constraints) to a "proxy agent", in a sealed-bid round**
- **The proxy agents bid in an underlying auction in "virtual time"**
- **Bidding in the underlying auction proceeds in a series of rounds, in which package bids are submitted**
- **After each round, *provisional winning bids* are determined that maximize revenues (in which bids stay "live" throughout the auction, and bids of a given bidder are mutually exclusive)**
- **The proxy agent's rule: when not a provisional winner, submit the allowable bid that, if accepted, would maximize the bidder's payoff (evaluated according to its reported values)**
- **The auction ends after a round with no new bids submitted by any of the proxy agents**

Results on Proxy Auctions

- **<u>Theorem</u>: The payoff vector resulting from the ascending proxy auction is in the core relative to the reported bidders' preferences**

- **<u>Theorem</u>: If π is a bidder-Pareto-optimal point in the core, then there exists a full information Nash equilibrium of the proxy auction with associated payoff vector π**
  - **Remark: These equilibria may be obtained using "semi-sincere" (or "profit-target") strategies: bid your true value minus a nonnegative constant on every package**

Pay-as-Bid Package Auction

- **A.k.a. "menu auction", the bidders submit package bids in a sealed-bid auction, the auctioneer determines the bids that maximize revenues, and the winning bidders pay the amounts of their bids**

- **<u>Theorem</u>: Under full information, the coalition-proof equilibrium allocations of the pay-as-bid package auction coincide with the bidder-optimal core allocations (Bernheim and Whinston, 1986)**

A Multiple Equivalence

- <u>Theorem:</u> **Suppose that the set V of possible bidder value functions includes all additive values. Then the following statements are equivalent:**
  - **The set V includes only values for which goods are substitutes**
  - **There exists a Walrasian equilibrium**
  - **For every profile of bidder valuations drawn from V, the seller's revenue in the Vickrey auction is monotonic**
  - **For every profile …, Vickrey payoffs are in the core**
  - **For every profile …, there is no profitable joint deviation by losing bidders nor a profitable shill-bidding strategy in the Vickrey auction**
  - **For every profile …, the pay-as-bid package auction has a unique full-information, coalition-proof equilibrium**
  - **For every profile …, the ascending proxy auction has the ex post equilibrium property**

Core-Selecting Auctions

- The underlying auction in “virtual time” can be suppressed, and the proxy auction can be reinterpreted as a sealed-bid package auction
- Moreover, the identified solution can be generalized to the notion of a “core-selecting auction” (Day and Raghavan 2007, Day and Milgrom 2008)
- Particular attention has been focused on the bidder-optimal core outcome that minimizes the Euclidean distance to the Vickrey outcome

Clock-Proxy Auction

- Since the proxy auction can be viewed as a sealed-bid auction, there is potentially a great information benefit in having a dynamic auction process lead up to it
- One approach is to have a clock auction followed by a final proxy auction round. The bids in the clock auction are interpreted as (binding) package bids; after the clock auction clears, bidders can submit additional package bids; and then the proxy auction is run based on all of the bids (Ausubel, Cramton and Milgrom 2006)

Open Issues / Directions

The Non-Substitutes Case

(1) What format(s) are best suited for auctions of heterogeneous items which are high in value and for which bidder preferences fail the substitutes condition?

(Leading example is telecom spectrum, but other important examples such as airport takeoff and landing slots also fit)

Package Bidding

(2) A Bayesian-Nash equilibrium analysis of a package bidding game with a rich private information structure

Multi-Unit Auctions

(3) Can anybody deliver an overwhelming theoretical or empirical argument for either the uniform-price or the pay-as bid auction, in environments where bidders have private information?

Dynamic Auctions

(4) There are at least two different roles attributed to the dynamic nature of many multi-item auctions (e.g. SAA and clock auction)

- The informational feedback of the dynamic auction may lead to higher revenues (Milgrom and Weber 1982) and/or greater efficiency; and
- It may narrow the relevant value reports that a bidder needs to make (simplifying the messages)

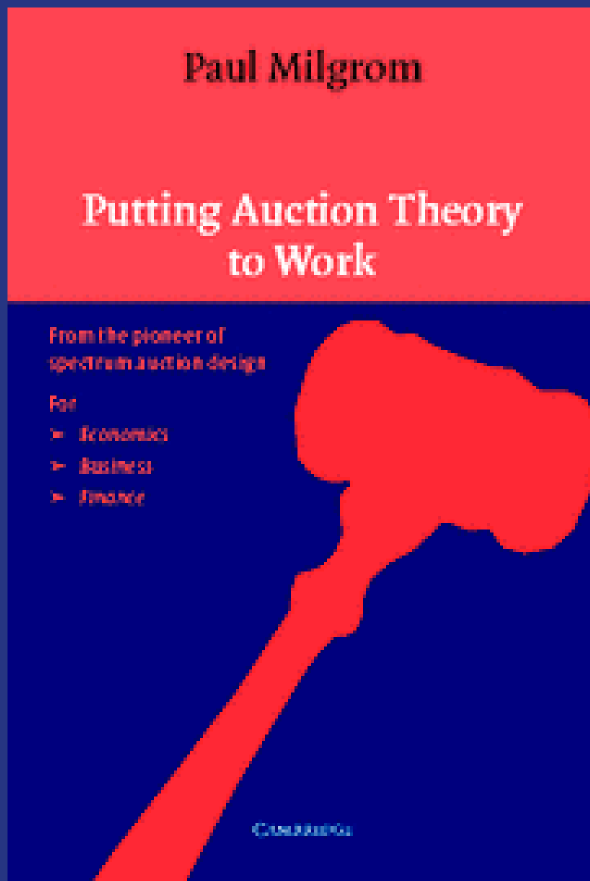
What is the relative importance of these two effects?

Exchanges

(5) Two-sided exchanges with package bidding open up a variety of difficulties, beginning with the possibility of a non-empty core. What progress can be made on the design of efficient exchanges?

Relationship with Matching

(6) There has been some initial success in bridging the auction literature and the matching literature (e.g. Hatfield and Milgrom 2005). Especially given the notion of a field of “market design” that includes each of these areas, it would be useful to develop further parallels and connections.


Paul Milgrom
Putting Auction Theory to Work
From the pioneer of spectrum auction design
For
Economics
Business
Finance