Lecture 4: Cognitive Imprecision and Strategic Interaction

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A Short Course on Cognitive Imprecision
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Strategic Interaction

- We have discussed in detail the idea that decisions are made in a way that is only **imprecisely** matched to the situation that someone is in (and the incentives provided by that situation)

  — and that **random noise** in cognitive processing gives choices a **stochastic** element, even conditional on a complete description of the situation and the DM's goals

- But we have so far mainly discussed **individual decision problems** — in which there may be uncertainty about the consequences that will follow from an action choice, but due to objective randomness in the external situation (and the probabilities may be **explicitly stated** by an experimenter)

Strategic Interaction

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- Today: discuss the consequences of cognitive imprecision when payoffs from action also **depend on others' action choices**

- Introduces a new reason for decisions to be **complex:** generally won't be trivial to decide how others will **choose,** even when you know their objective situation

  — and can't quantify this uncertainty using probabilities **stated by an experimenter**

- Also introduces a further reason (beyond those discussed in Lecture 1) for it to matter whether cognitive noise is **"early noise" or "late noise"**

Does the Nature of Cognitive Noise Matter?

- In (even very simple) **strategic** settings, there is an important difference between the two types of cognitive noise:

  — only comparison noise $\Rightarrow$ no problem recognizing the decision situation $\Rightarrow$ should be **common knowledge** what the game is [if no "private information"]

  — noisy representation of situation $\Rightarrow$ breaks common knowledge, **complicating coordination** of behavior, even when taken for granted that decision rules are **optimal**

A Simple (Linear-Quadratic-Gaussian) Example

- Consider a game with a **continuum** of players, who each simultaneously choose an action p (arbitrary real number, perhaps log of price set for individual differentiated good)
  - each player's payoff depends on own action p, **average** action $\bar{p}$ chosen by all players, and an exogenous **"fundamental"** s

A Simple (Linear-Quadratic-Gaussian) Example

- Suppose that each player's payoff is equal to

$$u(p, \bar{p}) \;=\; -\frac{1}{2}(p - s)^2 \;-\; \frac{\gamma}{2}(p - \bar{p})^2,$$

where parameter $\gamma > 0$ measures the degree of **strategic complementarity**

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where parameter $\gamma > 0$ measures the degree of **strategic complementarity**

- Unique **Nash equilibrium** (with common knowledge of the fundamental s):

$$p \;=\; s \quad \text{for all players}$$

regardless of the size of γ

Quantal Response Equilibrium

- At least in the lab, though, apparently **random** behavior is common

  — even in games like this, where Nash equilbrium would not require **mixed strategies**

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- The most common approach to modeling such behavior has been **"quantal response equilibrium"** (McKelvey and Palfrey, 1995)

  — instead of assuming that each player chooses a probability distribution over actions that is **optimal** (in the sense of maximizing expected payoff) given the prob. dist. over actions chosen by the **other** players [the requirement for **Nash equilibrium**], each player chooses each possible action with a probability that is an **increasing function** of the expected payoff from that action, given the prob. dist. over actions chosen by the other players

Quantal Response Equilibrium

- Thus QRE is a generalization of NE that introduces **comparison noise** into the conception of how each player's prob. dist. over actions is adapted to the incentives provided by the eq'm play of the other players — as in the individual decision problems discussed in Lecture 1

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- Has been successfully used to explain systematic discrepancies between observed play and NE predictions, in a variety of types of experimental games (Goeree *et al.*, 2016)

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- Has been successfully used to explain systematic discrepancies between observed play and NE predictions, in a variety of types of experimental games (Goeree *et al.*, 2016)

- But allowance only for comparison noise is not an innocuous assumption

The LQG Example Continued

- A common quantitative implementation of QRE: assume a **multinomial logit** model of choice by each player

  — **probability** of playing any action a in equilibrium is proportional to $e^{\lambda u(a)}$, where $\lambda > 0$ measures the (finite) precision of choice, and $u(a)$ is evaluated using the equilibrium probabilities of play by the other players

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- In our example, this implies that each player's action p will be a **random variable** with distribution

$$p \;\sim\; \exp(\lambda[-\frac{1}{2}(p-s)^2 \;-\; \frac{\gamma}{2}(p-\bar{p})^2])$$

The LQG Example Continued

- Or equivalently:

$$p \sim N\left(\frac{s + \gamma \bar{p}}{1 + \gamma}, \frac{1}{\lambda(1 + \gamma)}\right)$$

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$$\bar{p} = \frac{s + \gamma \bar{p}}{1 + \gamma} \quad \Rightarrow \quad \bar{p} = s$$

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- Unique QRE distribution of actions:

$$p \;\sim\; N\left(s,\; \frac{1}{\lambda(1+\gamma)}\right)$$

— strategic complementarity (γ) affects the **dispersion** of individual actions, but not the **aggregate** response to changes in s

The LQG Example Continued

- Suppose instead that each player must base their action on an individual-specific **noisy internal representation** of the publicly-observable state s

$$r = s + \epsilon, \qquad \epsilon \sim N(0, \nu^2)$$

and that s is drawn from a **prior** [to which we will assume individuals' decision rules have adapted]

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- If in equilibrium, aggregate response is given by a function $\bar{p}(s)$, these specifications define a **joint distribution** for $(s, r, \bar{p})$, and the **optimal decision rule** for an individual in this environment will satisfy

$$p(r) \;=\; \frac{1}{1+\gamma}\mathrm{E}[s\,|r] + \frac{\gamma}{1+\gamma}\mathrm{E}[\bar{p}(s)\,|r] \quad \forall r$$

The LQG Example Continued

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- Consistency then requires that $\bar{p}(s) = \mathrm{E}[p(r)\,|s] \;\; \forall s$

- If we conjecture a solution of the form $\bar{p}(s) = p_0 + \xi s$, the optimal decision rule must be

$$p(r) \;=\; \frac{1+\gamma\xi}{1+\gamma}[\mu + \beta(r-\mu)] + \frac{\gamma}{1+\gamma}p_0$$

where

$$\beta \;\equiv\; \frac{\sigma^2}{\sigma^2+\nu^2} \;<\; 1, \qquad \text{so that}$$

$$\bar{p}(s) \;=\; \frac{1+\gamma\xi}{1+\gamma}[\mu + \beta(s-\mu)] + \frac{\gamma}{1+\gamma}p_0$$

The LQG Example Continued

- Equating coefficients, we obtain a unique solution for p_0 and ξ, implying that

$$p \sim N(\mu + \xi(s - \mu),\ \xi^2 \nu^2)$$

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- Thus an optimal adaptation to "early noise" implies **reduced sensitivity** of the aggregate action $\bar{p}$ to variation in s, relative to the NE prediction [another example of "behavioral attenuation"]

— moreover, the attenuation is **greater** (ξ is smaller) the greater the degree of **strategic complementarity** (i.e., the larger is γ)

A Macroeconomic Application

- A possible interpretation of the game: s represents exogenous variation in (the log of) **aggregate nominal spending,** p is (the log of) each producer's **price,** and $u(p, \bar{p})$ indicates how the producer's profits depend on its own price and the prices of other differentiated goods

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- In the NE, prices p all perfectly track variation in $s \Rightarrow$ **no real effects** of monetary disturbances that cause **nominal** spending to vary

  — and introducing noise via QRE **doesn't change this** (prices are dispersed, but each increases by exactly the amount of any increase in s)

A Macroeconomic Application

- "Early noise" instead allows monetary shocks to have **real effects** (especially if strong SC)

  — get "hump-shaped" response to a permanent increase in nominal spending, in a dynamic extension of the model (Woodford, 2003; Melosi, 2014)

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- Unlike the imperfect-info model of Lucas (1972), this model doesn't require that the monetary shock **not be publicly observable,** a particular problem for explaining **persistent** real effects using the Lucas model

Coordination in the Lab

- Game studied experimentally by Frydman and Nunnari (2025):

	leave	*stay*
leave	(θ, θ)	$(\theta, 47)$
stay	$(47, \theta)$	$(63, 63)$

- each of two players must **simultaneously** make a binary decision
- payoffs for each depend on their **joint** decision
- parameter θ is different on different trials

Nash Equilibrium

- We look for an equilibrium in **decision rules,** specifying each player's action (or probability distribution over actions) as a function of the state θ on that trial

  — each player's decision rule is a **function** $p(\theta)$ indicating prob($stay\ |\theta$)

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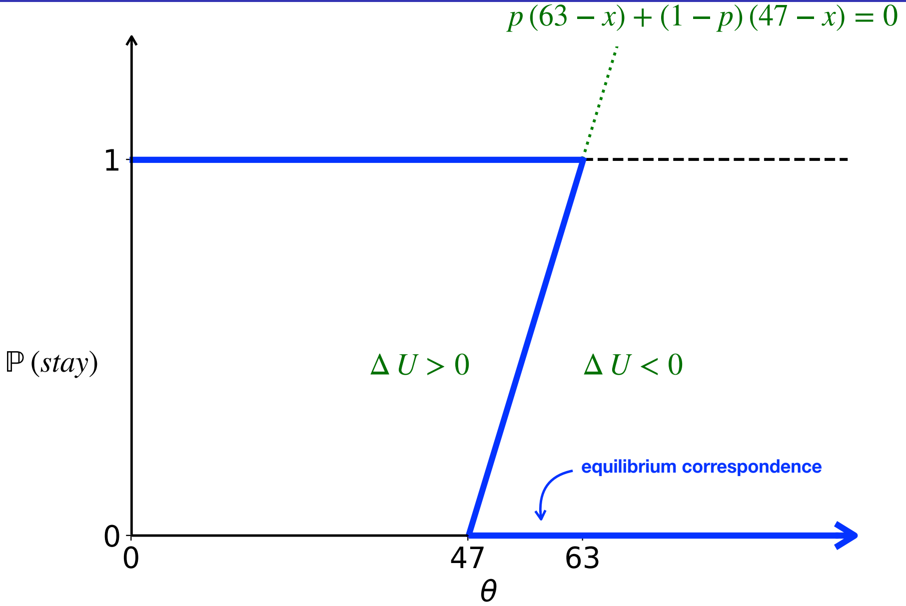
- First step: consider the **correspondence** consisting of all pairs (θ, p) with the property that when the state is θ, prob($stay$) $=$ p is a **best response** to prob($stay$) $= p$ on the part of the other player

The Equilibrium Correspondence



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- Thus theory predicts that behavior should **change discontinuously** with changes in the state

  — moreover, the location of the jump (or jumps) is **indeterminate**

- Equilibrium in **threshold strategies** ($p = 1$ for all $\theta \leq \theta^*$, $p = 0$ for all $\theta > \theta^*$): this is a symmetric eq'm for **any** $\theta^* \in [47, 63]$

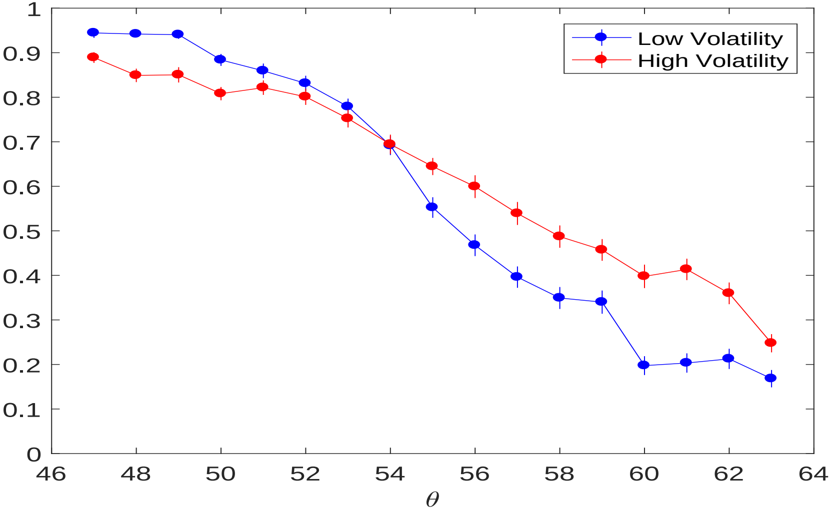
Experimental Evidence

- Experiments (e.g., Heinemann *et al.*, 2004, 2009; Frydman and Nunnari, 2025):
    - $p(\theta)$ not always either **zero** or **one** — instead, wide range of values of θ for which behavior appears to be **probabilistic**

Experimental Evidence

- Experiments (e.g., Heinemann *et al.*, 2004, 2009; Frydman and Nunnari, 2025):
  - $p(\theta)$ not always either **zero** or **one** — instead, wide range of values of θ for which behavior appears to be **probabilistic**
  - no obvious **jumps** in action probability as state varies — instead, prob. of staying gradually declines for larger θ
  - intermediate probabilities are lower for larger θ, rather than **increasing** in θ as would be required for a selection from the EC

Frydman and Nunnari (2025)



vertical axis = probability of staying

horizontal axis = θ

A Model of Financial Crises?

- It is sometimes thought to be a virtue of this kind of model that it implies that equilibrium should be **indeterminate** — so that it is consistent with eq'm behavior for collective behavior to shift **abruptly,** without any change in economic "fundamentals" having had to occur

  — offered as an explanation for the **sudden onset** of financial crises [e.g., "second-generation" models of speculative attack on a currency peg (Obstfeld, 1996)]

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- But the model doesn't explain fact that deteriorating fundamentals **do** seem important for predicting when crises occur (Gorton, 1988, 2012), as opposed to the two NE being equally possible **anywhere** in a wide range of values for θ

Another Problem with the NE Prediction

- In above analysis, the eq'm correspondence is the same regardless of the **distribution** from which θ is drawn on different occasions

  — hence no reason for the **equilibrium strategy** $p(\theta)$ to differ across environments associated with different ranges of variation in θ

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- But in Frydman and Nunnari (2025) experiment: two different treatments ["high volatility" and "low volatility"] that differ only in the **variance** of the distribution from which θ is drawn (independently on each trial)

  — mean of θ is same in both cases: 55, the midpoint of the "indeterminacy range" on the NE correspondence

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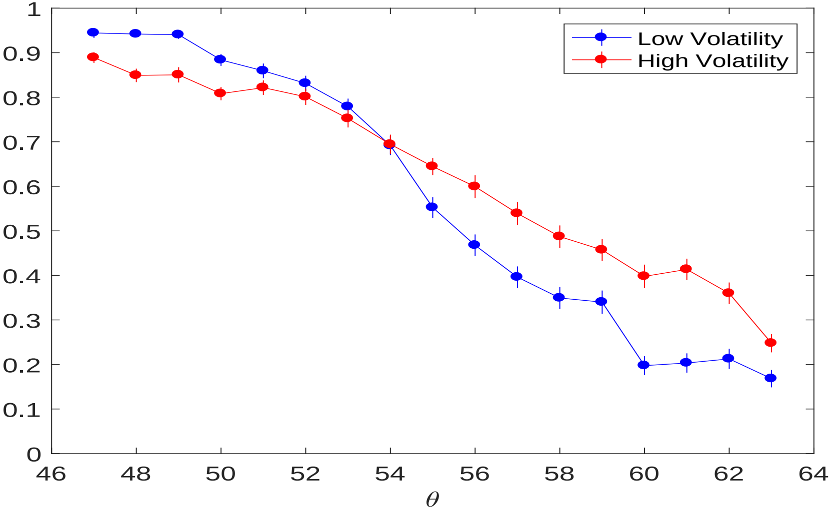

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Allowing for Cognitive Imprecision

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- If we define the expected payoff **differential** from staying vs. leaving, as a function of the other player's prob. p of staying,

$$\Delta(p) \equiv u^{stay}(p) - u^{leave} = (47 - \theta) + 16p,$$

then for the game defined by any value of θ, a symmetric **Nash equilibrium** is a probability of staying p^* (for both players) with the property that

$$(\Delta(p^*),\, p^*) \in \mathcal{C}^{Nash}$$

where $\mathcal{C}^{Nash}$ is the correspondence of values (Δ, p) such that p is an optimal choice in the case of expected payoff differential Δ [graphed as black segments on slide 28]

Quantal Response Equilibrium

- Introducing **comparison noise** simply changes the correspondence $\mathcal{C}^{Nash}$ to $\mathcal{C}^{QRE}$, the graph of the function

$$p^* = \Phi(\Delta(p^*))$$

indicating the probability of staying as an increasing function of expected payoff differential

— for example, $\Phi(\Delta) \equiv \frac{e^{\Delta/\phi}}{e^{\Delta/\phi}+1}$, where $\phi > 0$ indexes the degree of noise [low noise $\leftrightarrow$ large ϕ]

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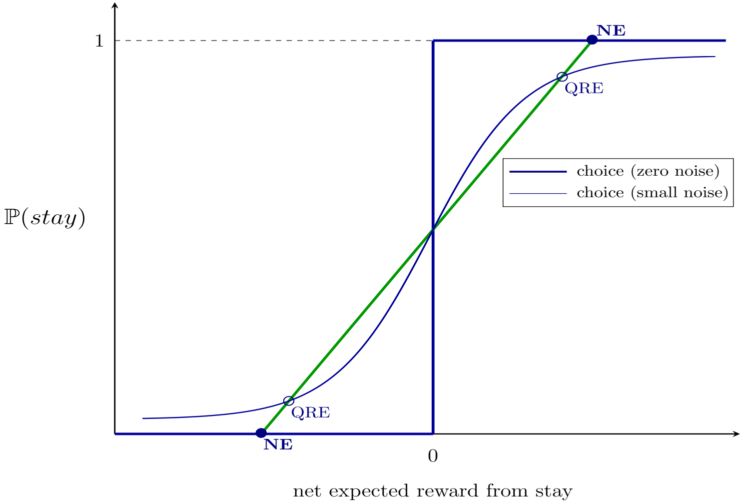
- Thus a **symmetric QRE** corresponds to a point of intersection of the **sigmoid curve** $p^* = \Phi(\Delta)$ and the **green line** $\Delta = \Delta(p^*)$ in the figure on the next slide

QRE Compared to NE (for given θ)



green line: graph of $\Delta(p)$

Quantal Response Equilibrium

- In the case of **small enough noise** [e.g., large enough ϕ in the parametric model of comparison noise proposed here], the graph of $\mathcal{C}^{QRE}$ will be close to $\mathcal{C}^{Nash}$, and hence will intersect the green line at points close to each of the intersections between the green line and $\mathcal{C}^{Nash}$

- Hence in the case of small enough noise, there will again be a **multiplicity of equilibria**

Quantal Response Equilibrium

- If we consider now the **equilibrium correspondence** graphing all QRE in the $p - \theta$ plane: in the case of small enough noise, shape will be a **backward S shape** — not too different from the Z-shaped correspondence on slide 18

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- If we consider now the **equilibrium correspondence** graphing all QRE in the $p - \theta$ plane: in the case of small enough noise, shape will be a **backward S shape** — not too different from the Z-shaped correspondence on slide 18

- Again any selection $p(\theta)$ from the EC must involve **at least one discontinuous jump**

  — not possible for $p(\theta)$ to gradually decline with increases in θ, as observed in experiments

  — and a broad range over which changes in "fundamentals" don't have any necessary implication for probability of a jump from high to low p

Quantal Response Equilibrium

- In addition, the set of QRE associated with each value of θ are **independent** of the distribution from which θ may be drawn on different occasions

  — hence no reason for the function $p(\theta)$ to differ across the two different treatments of Frydman and Nunnari (2025)

Optimal Adaptation to "Early Noise"

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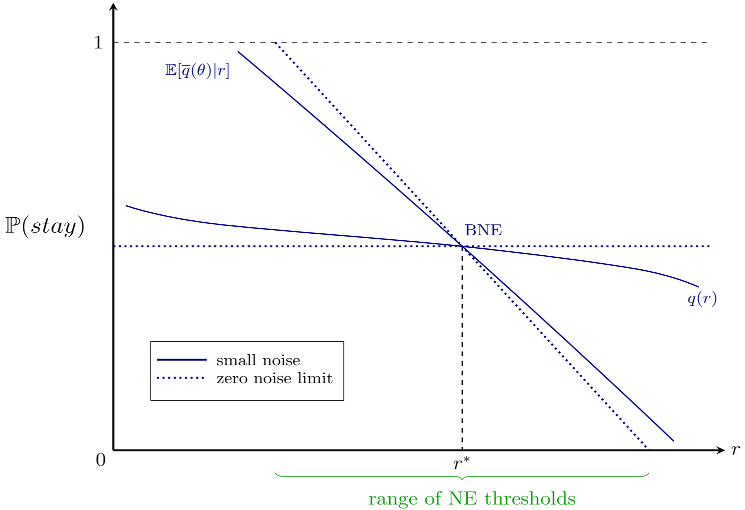
- **Equilibrium:** each player leaves if and only if $r > r^*$ (for them), where each player's threshold r^* is the point at which expected payoff from leaving is exactly equal to expected payoff from staying [given other player's decision rule, and optimal Bayesian decoding of the noisy representation r]

Determination of Equilibrium r^*



$\bar{q}(\theta) =$ required prob. other stays, to make it optimal to stay (if state θ)

$$q(r) \ = \ \mathrm{P}[r_{-i} < r \, | r_i = r]$$

Determination of Equilibrium r^*

- In the limit as cognitive noise of this kind becomes negligible:
  - $q(r)$ approaches constant value $1/2$ for all r [horizontal dashed line in figure: other's internal state equally likely to be above or below one's own]
  - $\mathrm{E}[\bar{q}(\theta)\,|r_i = r]$ approaches $\bar{q}(\theta(r))$ [downward sloping straight line, as shown, if $m(\theta)$ is linear]

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- Hence in the case of **small enough** cognitive noise of this kind, the intersection must be **unique,** as shown
  - set of solutions for r^* as $\nu \to 0$ [single point labeled BNE] **not** the same as the set of solutions for r^* in the zero-noise model [entire interval marked by the curly bracket]

Optimal Adaptation to "Early Noise"

- Result: if cognitive noise is **small enough** (though non-zero), there will be **unique** equilibrium strategies

  — unique threshold r^* such that r^* is optimal choice for a player, given that other uses threshold r^*

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- Moreover, the unique equilibrium prediction is that the probability that players stay should be a **decreasing function of** θ [even for values of θ in the range where there are multiple NE]

- This prediction of **sensitivity to fundamentals** is more consistent with behavior observed in laboratory experiments, and arguably with what is observed in real-world financial crises

"Global Games"

- This uniqueness result [in the limiting case of negligible, but non-zero, noise variance] has been stressed in the literature on "global games" (Morris and Shin, 1998, 2003)

"Global Games"

- This literature generally supposes that, instead of players observing the state θ, they each only have (independent) "private signals" about its value

  — result is that even when each agent's private signal is extremely (but not perfectly) precise, one gets uniqueness; taken to imply fragility of the full-information analysis

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- But literature has stressed that result relies on assumption that there are not **also "public signals"** — and that there should continue to be multiple equilibria as long as public signals (while also imperfect) are **sufficiently informative** relative to the informativeness of the private signals (e.g., Angeletos and Werning, 2006; Hellwig *et al.*, 2006)

"Global Games"

- This discussion assumes that people perfectly observe whatever is visible to them — and that they can be sure that others also perfectly observe whatever they know that those others can see, and so on

  — imperfect observability of θ must reflect a fact about the structure of the external world

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  — imperfect observability of θ must reflect a fact about the structure of the external world

- If instead the imprecise internal representation r represents a **cognitive constraint,** the mere existence of a variable (e.g., a market price) that everyone can observe does not create a "public signal" in the sense assumed in this literature

  — there will not be **common knowledge** that everyone else must observe the "public signal" in exactly the same way that you do

Explaining Context-Dependence

- In symmetric eq'm, the players' probability of staying should be given by:

$$\text{Prob}(\text{stay}\,|\theta) \;=\; \text{Prob}(r_i < r^*\,|\theta) \;=\; \Phi\left(\frac{r^* - m(\theta)}{\nu}\right),$$

where now $\Phi(z)$ is the CDF of the standard normal distribution.

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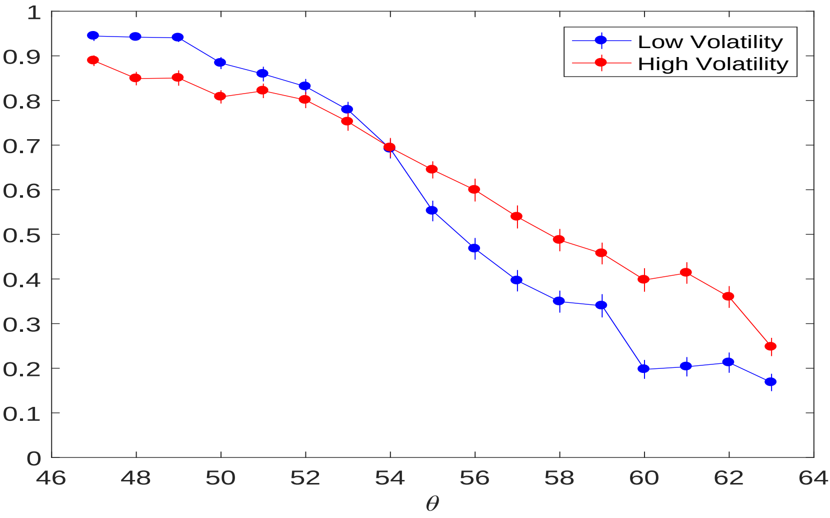
- This should be a decreasing function of θ (as in F&N data).

- And if when the range of values of θ in the support of the prior is **wider,** the encoding function $m(\theta)$ must be **flatter** [range normalization], then the function $\text{Prob}(\text{stay}\,|\theta)$ should also be a **flatter** function of θ (also as in F&N data).

Frydman and Nunnari (2025)



vertical axis = probability of staying

Rational Inattention?

- Can the kind of **context-dependence** of the degree of sensitivity of players' behavior to variations in θ that F&N document be explained by a Sims-style model of rational inattention?

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- No — Yang (2015) analyzes eq'm in this kind of game with RI players, and shows that for any small enough information cost parameter,
  - equilibrium is **indeterminate:** even if one restricts attention to symmetric equilibria in which $p(\theta)$ is monotonically decreasing, there is a continuum of such equilibria
  - and any equilibrium necessarily involves a **discontinuous jump** — the location of which can be any value of θ over a wide interval

Rational Inattention?

- Why the difference? RI doesn't imply the kind of noisy internal representation assumed by F&N (or the global games literature)

  - according to RI, efficient representation conveys no more information than **which action is optimal** given the current value of θ [and opponent's decision rule]

  - internal representation is a binary state, and conditional probability of the "stay" state **jumps discontinuously** at some critical value θ^*

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    - according to RI, efficient representation conveys no more information than **which action is optimal** given the current value of θ [and opponent's decision rule]
    - internal representation is a binary state, and conditional probability of the "stay" state **jumps discontinuously** at some critical value θ^*
    - equilibrium concept allows the two players to **coordinate** on the location of this jump (as in the full-info NE analysis)

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- Instead, F&N assume that internal representation is optimized only within a **more restrictive class** of possibilities — motivated by **efficient coding** models from computational neuroscience [for related proposals, see also Hébert and Woodford, 2021; Morris and Yang, 2022; Aridor *et al.*, 2025]

  — this arguably leads to more realistic conclusions in this economic application

Summary

- The hypothesis that decisions are based on **noisy internal representations** of the presented data can explain phenomena that a mere assumption of **comparison noise** (or more generally, response noise) cannot
  - especially when the hypothesis of noisy coding is combined with the further assumptions of **efficient coding** and **Bayesian decoding** [often used in the literature on perceptual errors]

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  — only that a hypothesis of comparison noise **by itself** doesn't adequately capture the role of cognitive noise in decision making
- Another illustration of how study of cognitive noise in perceptual domains can help to improve economic modeling

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