The Elasticity of Substitution between Land and Capital:
Evidence from Chicago, Berlin, and Pittsburgh

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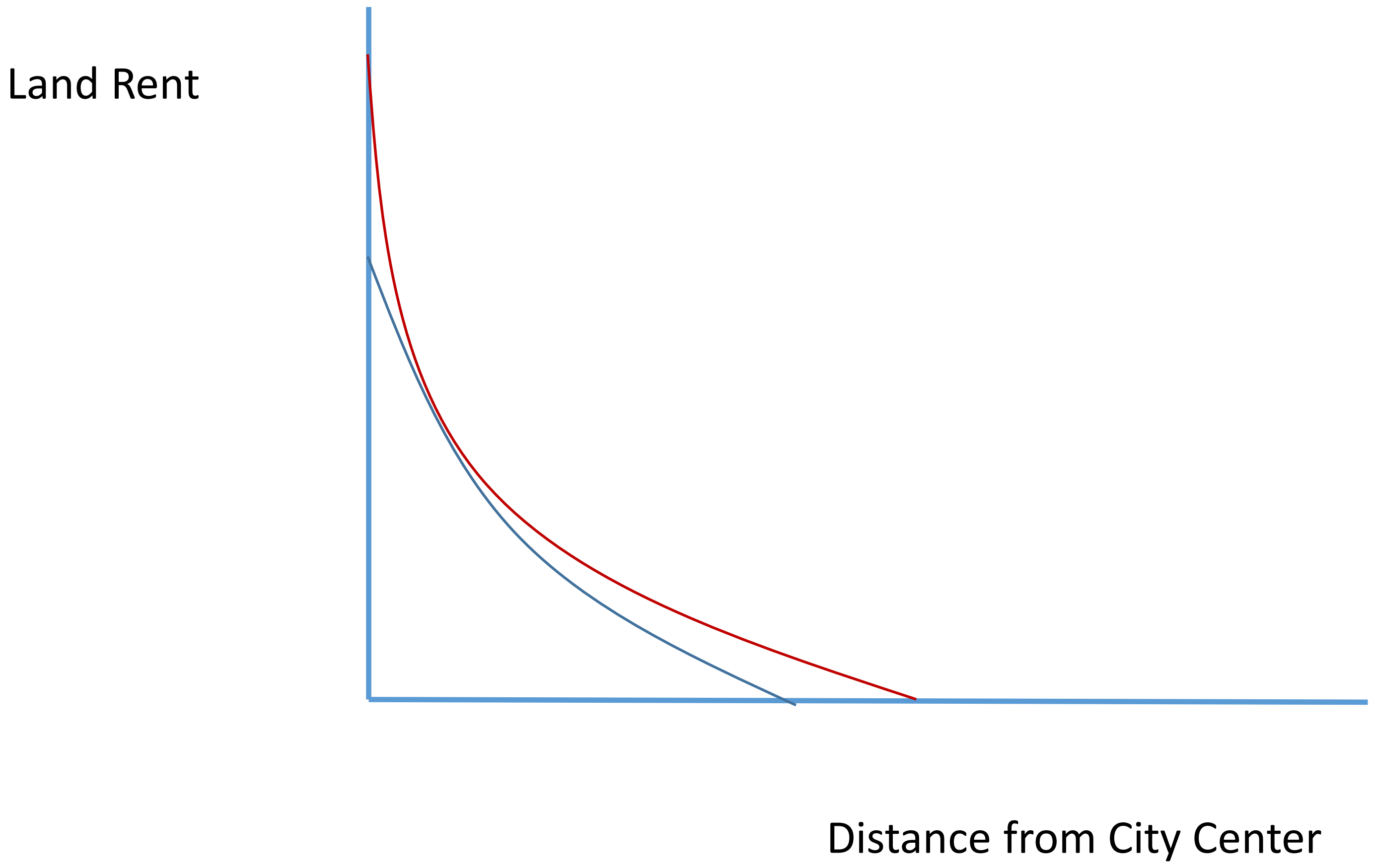
Implications of the Elasticity of Substitution



Higher elasticity implies greater ability to substitute capital for land in production – taller buildings on smaller lots as land rent increases

Firm location choices are also determined by the elasticity – more likely to be in city center if elasticity is higher

Influences from Leon Moses

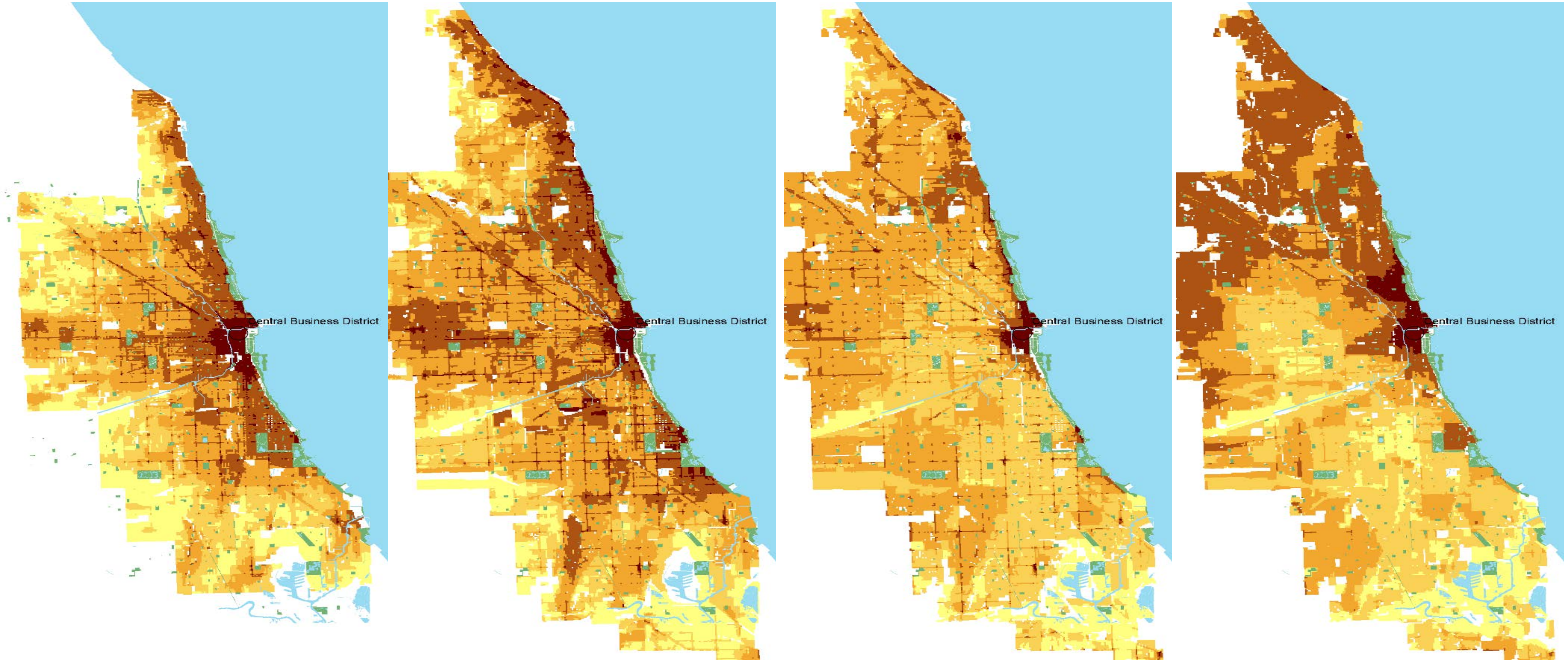
- "Location and the Theory of Production," Quarterly Journal of Economics (1958). "Objective is to place theory of location within the main body of economic theory," and "to investigate the implications of factor substitution for the locational equilibrium of the firm."
- "Land-Use Theory and the Spatial Structure of the Nineteenth-Century City," *Papers in Regional Science* and *Spatial, Regional, and Population Economics: Essays in Honor of Edgar Hoover*, with Raymond Fales (1972).
  - Locations of 659 Chicago manufacturing firms in 1873, just after the fire.
  - Employment was remarkably decentralized even then.
  - Transportation costs were a primary determinant of firm locations – tradeoff between access to input markets and final market. (Bricks near the source of clay along the river; beer along the lake – ice; slaughtering near rail.)

Objective: "Extend the Weberian model in ways that can help explain the distribution of all industries rather than individual ones."

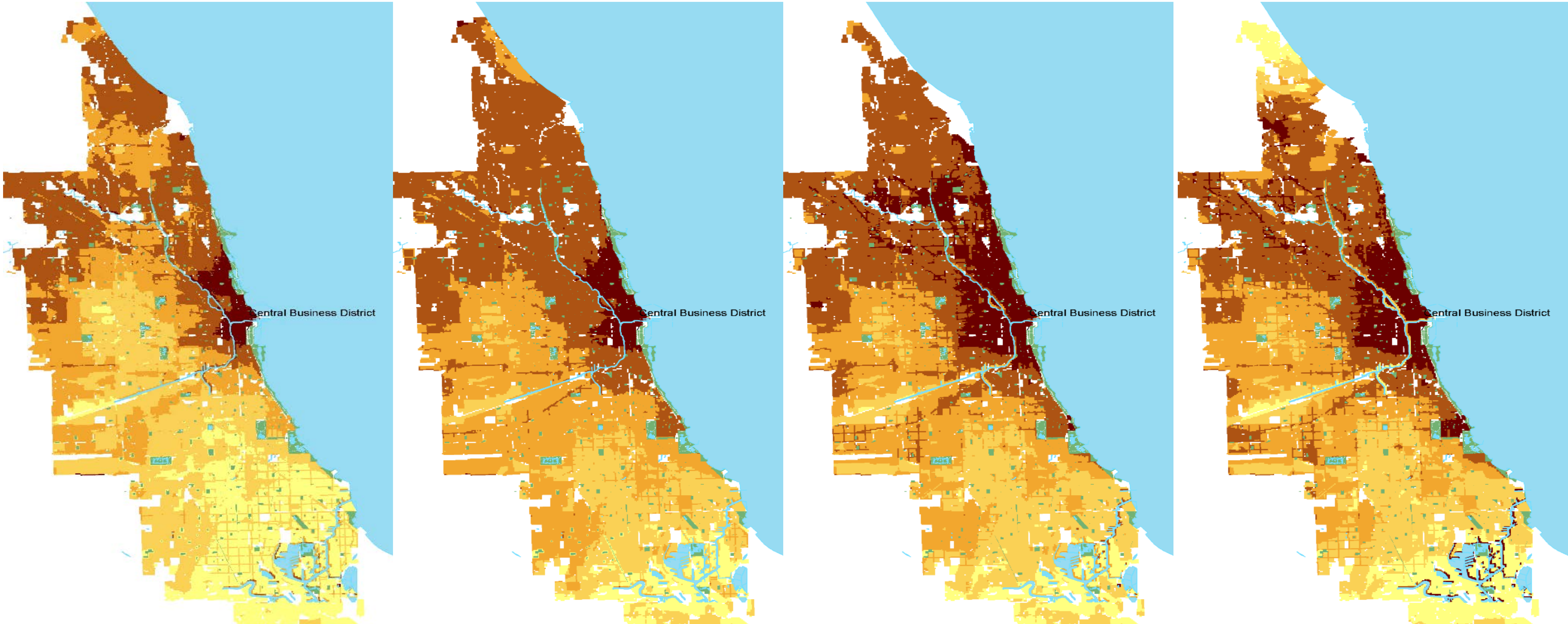
1. *Scale economies in interregional transport were very great*.
2. *Intra-urban freight transport was less technologically developed*. "Process requiring large amounts of weight-losing materials that were available locally would tend to be drawn to the sites of these materials."
3. Materials orientation may have been more important than market orientation. Many industries were "weight-losing"
4. *Intra-urban person transport was efficient relative to freight transport*.
5. *A gap also existed in the technology of information flow*. Firms oriented toward information clustered near telegraph terminals.

Land Values in Chicago, 1913, 1939, 1965, 1990

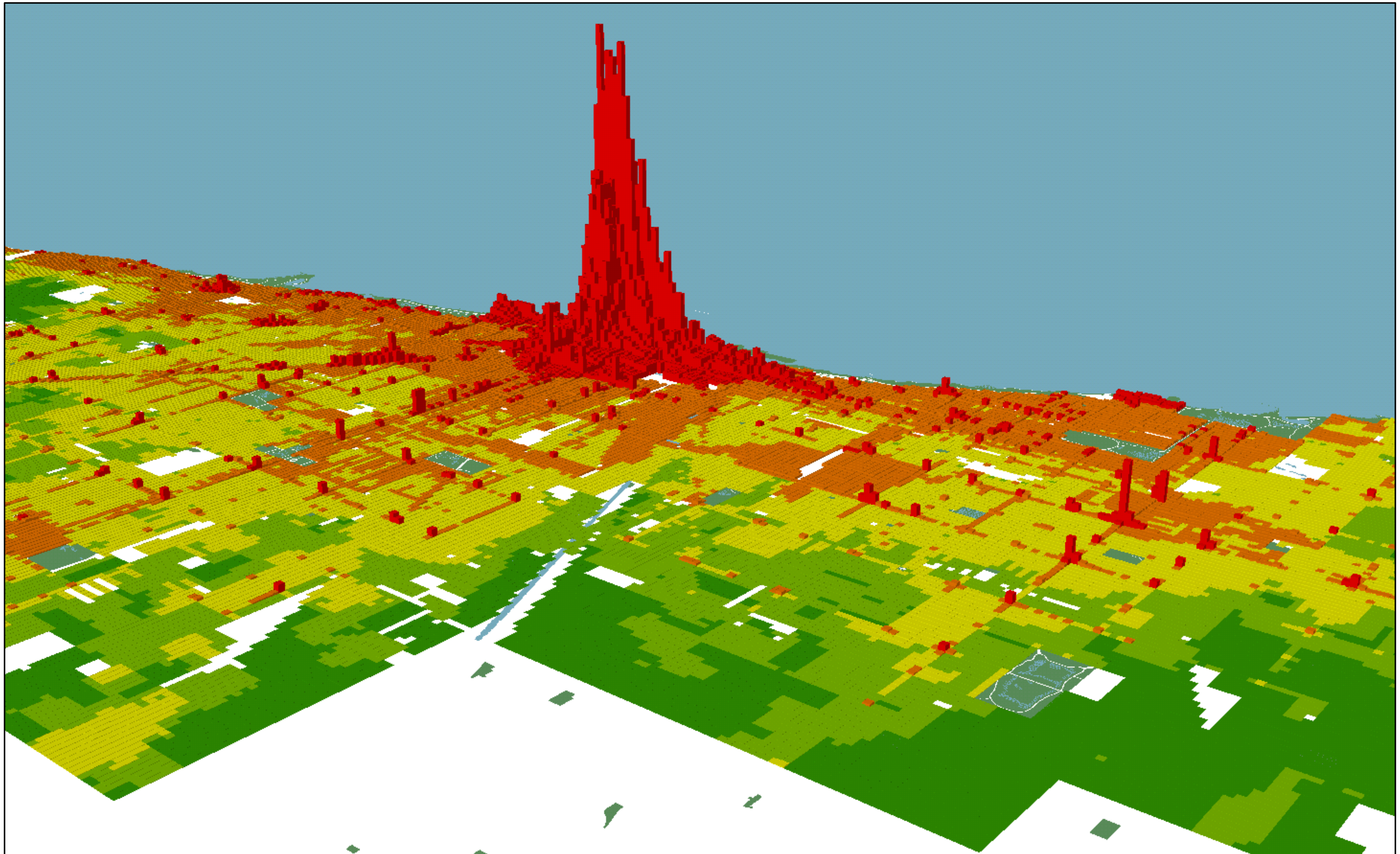
(with Gabriel Ahlfeldt, LSE) Source: Olcott’s Land Values Blue Book of Chicago



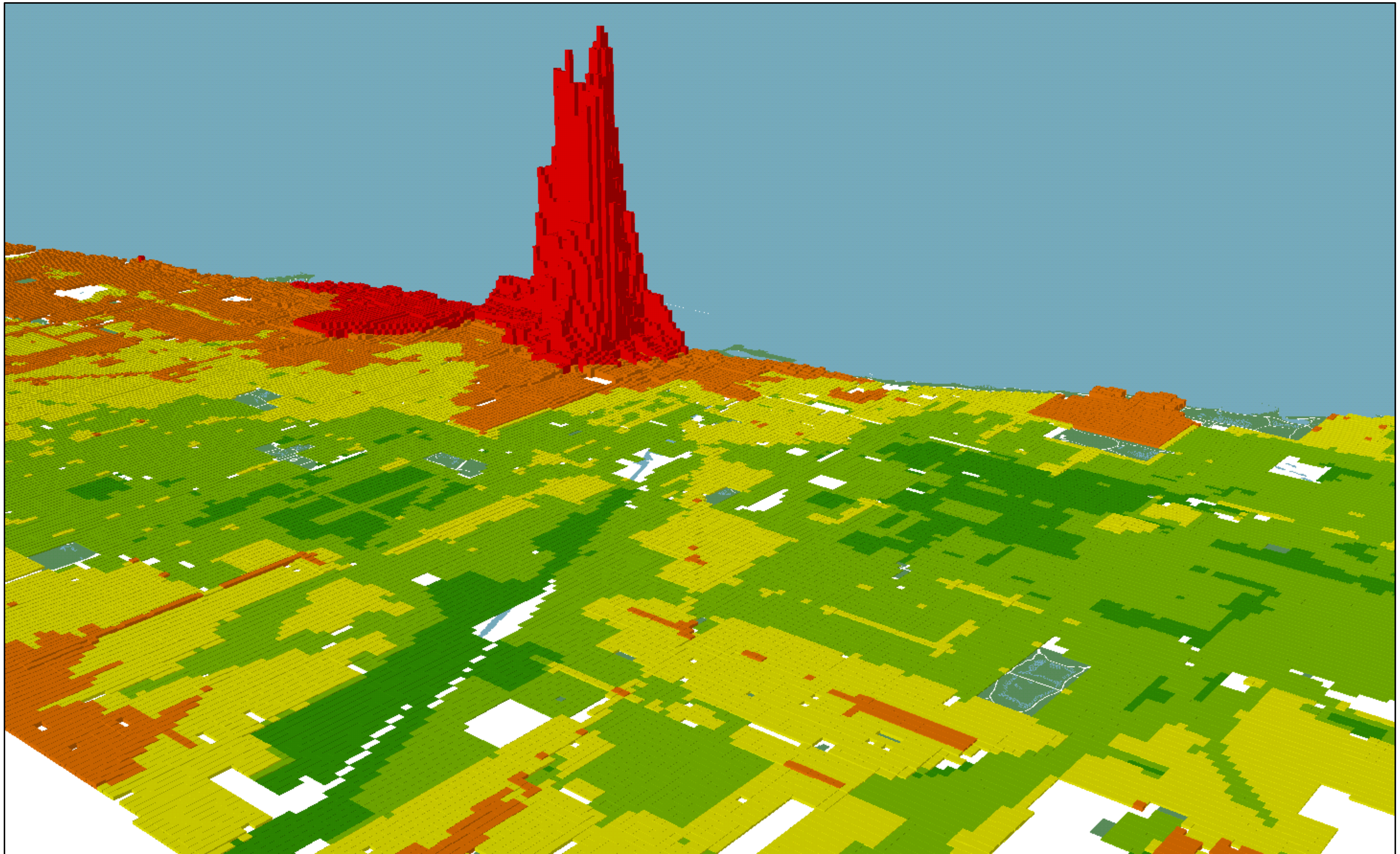
Land Values in Chicago, 1995, 2000, 2005, 2009
Source: Vacant Land Sales



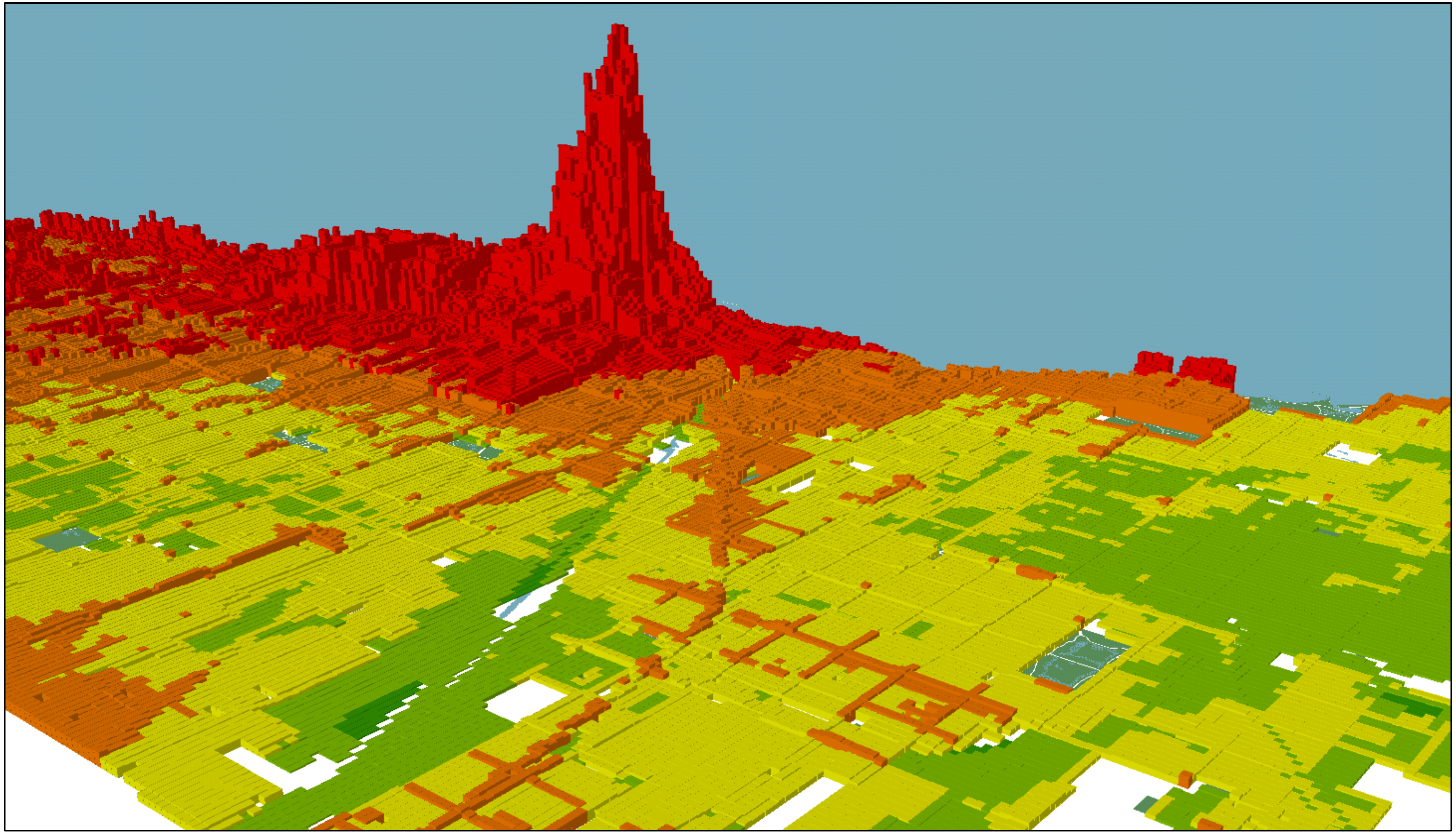
Land Value Surface, 1913



Land Value Surface, 1990



Land Value Surface, 2005 (Vacant Land Sales)



Estimating the Elasticity of Substitution between Land and Capital in the Production of Housing
(with Gabriel Ahlfeldt, LSE)

- Classic approach

$$log\left(\frac{K}{L}\right) = c + \sigma logR,$$

where K = capital, L = land, R = land rent. σ = elasticity of substitution.

K is not observed. Do observe house sale price (PH), lot size L, and R.

$$log\left(\frac{pH - RL}{L}\right) = c + \sigma logR$$

Problem: Measurement error in R may lead to downward bias in estimated elasticity. Good instruments are not necessarily available.

Conclusion: Elasticity of approximately 0.6? Range of about 0.4 – 1

Epple, Gordon, and Sieg, "A New Approach to Estimating the Production Function for Housing," (AER, 2010)

- Under the assumption of a concave, constant returns to scale production function and a competitive construction sector, EGS show that land value is a function of housing value per unit of land:

$$R = f(v)$$

where v = PH/L   (House value per unit of land)

- Implication for capital – land ratio:

$$\frac{K}{L}(v) = v - R(v)$$

- By definition:

$$\sigma = \frac{dlog(K/L)}{dlogR}$$

Estimation Procedure

- 1. Nonparametric estimation of $R = f(v)$
- 2. Second stage estimation to calculate $\widehat{\sigma}$
  - a. Regression

$$\log\left(\frac{\widehat{K}}{L}\right) = log\left(v - \hat{R}(v)\right) = \kappa + \sigma log\hat{R}(v)$$

  - b. Directly calculate from first-stage estimates. $\sigma = \frac{dlog(K/L)}{dlogR}$ implies:

$$\hat{\sigma} = \frac{\hat{f}(v)}{v - \hat{f}(v)}\left(\frac{1}{\hat{f}'(v)} - 1\right)$$

Alternative Estimation Procedure with Log-Log Form

- 1. Nonparametric estimation of $logR = g(\log(v))$
- 2. Second stage estimation to calculate $\widehat{\sigma}$
  - a. Regression

$$\log\left(\frac{\widehat{K}}{L}\right) = log(v - \exp(\hat{g})) = \kappa + \sigma log\hat{R}(v)$$

  - b. Directly calculate from first-stage estimates. $\sigma = \frac{dlog(K/L)}{dlogR}$ implies:

$$\hat{\sigma} = \frac{1}{v - exp(\hat{g}(v))}\left(\frac{v}{\hat{g}'(v)} - exp(\hat{g}(v))\right)$$

Some Monte Carlo Results

	σ = .5	σ = .25	σ = 1	σ = 1.25	$R = f(v)$, $\bar{\delta} = 1.26$	$logR = g(\log(v))$ $\bar{\delta} = 1.14$
OLS	0.141	0.308	0.477	0.648	0.708	0.722
	(0.018)	(0.018)	(0.020)	(0.023)	(0.023)	(0.018)
IV, cor(Z, e) = 0	0.472	0.741	1.012	1.273	1.232	1.260
	(0.028)	(0.037)	(0.047)	(0.055)	(0.052)	(0.048)
IV, cor(Z, e) = 0.50	0.186	0.370	0.555	0.742	0.778	0.978
	(0.019)	(0.021)	(0.024)	(0.030)	(0.024)	(0.040)
1: Linear LWR 2: Regression	0.496	0.753	1.004	1.234	1.205	1.127
	(0.020)	(0.028)	(0.035)	(0.041)	(0.078)	(0.024)
Single-Stage Linear LWR	0.525	0.762	1.006	1.267	1.438	1.350
	(0.033)	(0.043)	(0.053)	(0.066)	(0.094)	(0.152)
1: Log-Log LWR 2: Regression	0.512	0.762	1.007	1.235	1.201	1.236
	(0.018)	(0.024)	(0.032)	(0.038)	(0.034)	(0.035)
Single-Stage Log –Log LWR	0.501	0.763	1.010	1.231	1.220	1.236
	(0.024)	(0.030)	(0.036)	(0.041)	(0.036)	(0.037)

Data

- Chicago
  a) 1990 Olcott's for R; house prices for sales of new homes, 1986-94. n = 414.
  b) Vacant land sales for R, 1983-2011; nonparametric regression to predict values for all homes that were built during this period. N = 3,576.

- Berlin

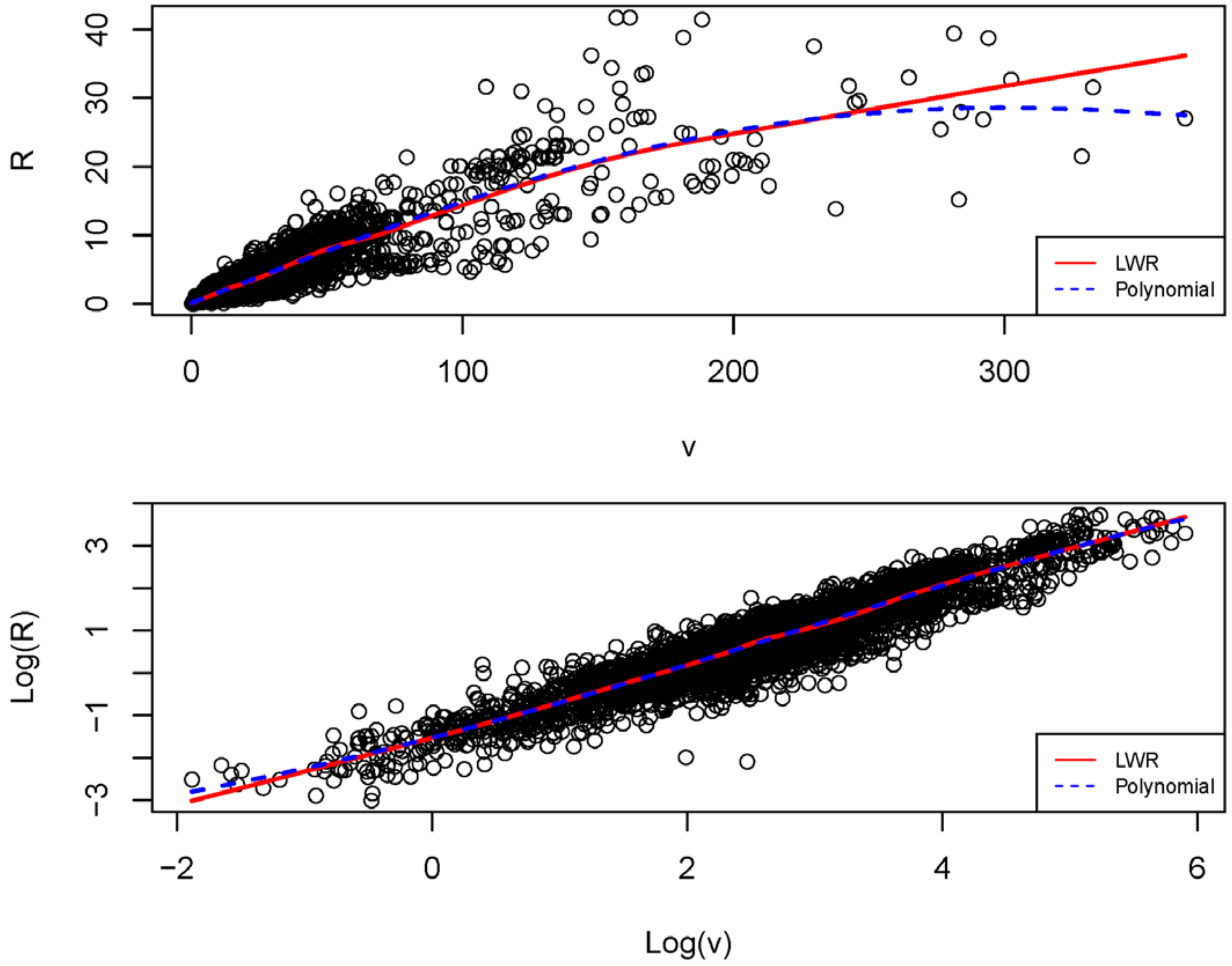
All sales of developed properties, 1990-2010. Assessed land values.

273 commercial properties, n = 5,466 for residential, no more than 5 years old.

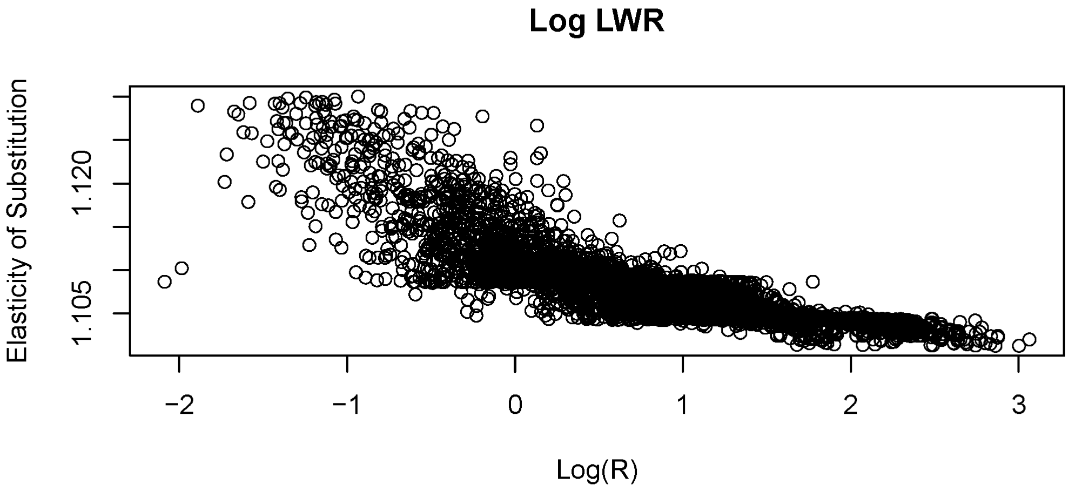
- Pittsburgh (Allegheny County)

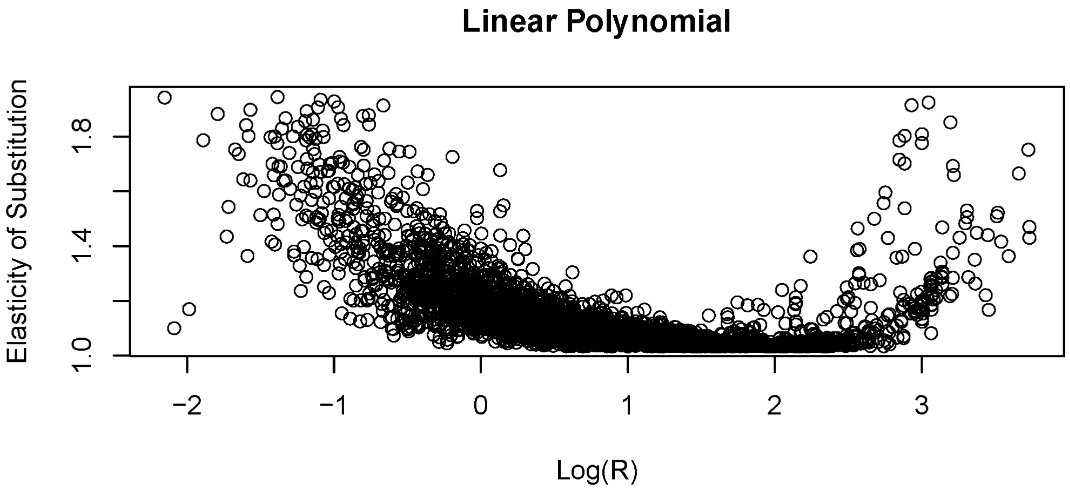
Assessments from 2001 for both land values and house prices. Homes built 1995 – 2001. 992 commercial properties, n = 6,362 for residential.

Raw Data and Estimates for Pittsburgh



Elasticity of Substitution Estimates for Pittsburgh

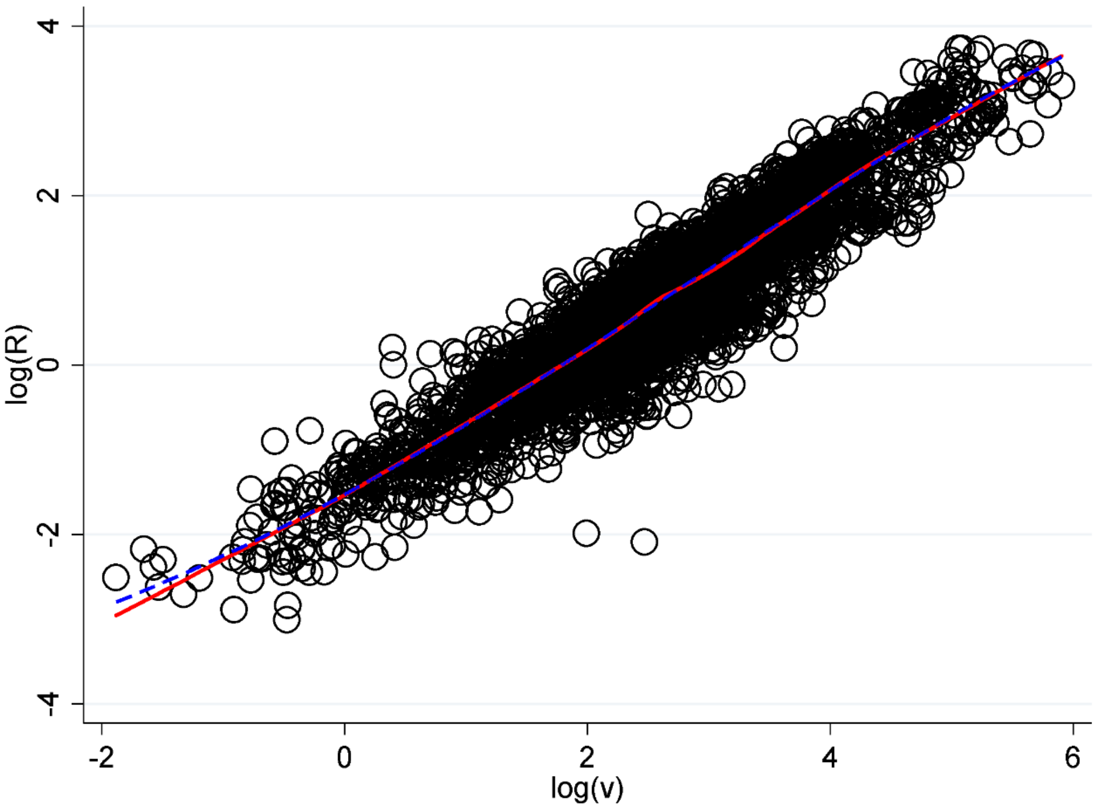




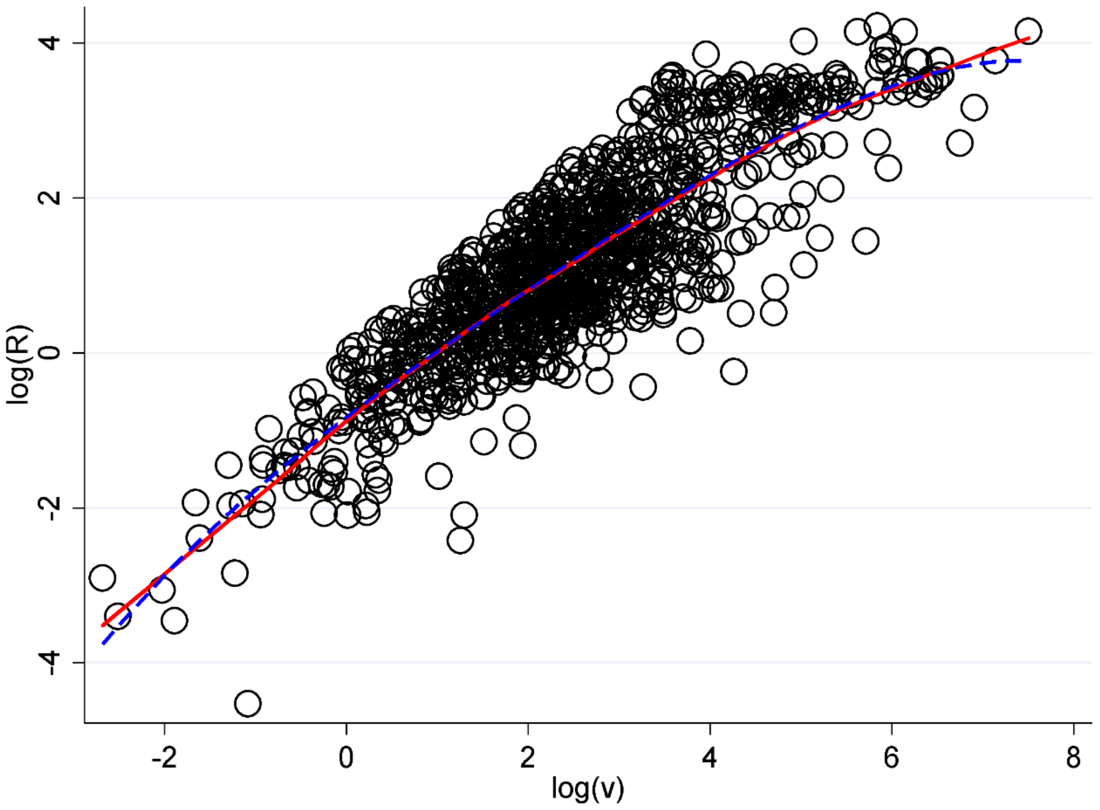
Mean Elasticities for Pittsburgh

	Regression (Two-stage)	Mean Elasticity (Single-stage)	Mean Elasticity, 1% - 99% Percentiles	Mean Elasticity, 5% - 95% Percentiles
4^{th}-Order Poly. R on v	1.175	1.228	1.140	1.110
LWR, R on v	1.132	1.234	1.216	1.204
4^{th} Order Poly., Log(R) on log(v)	1.119	1.104	1.100	1.093
LWR, log(R) on log(v)	1.119	1.109	1.108	1.108

Estimates for Pittsburgh

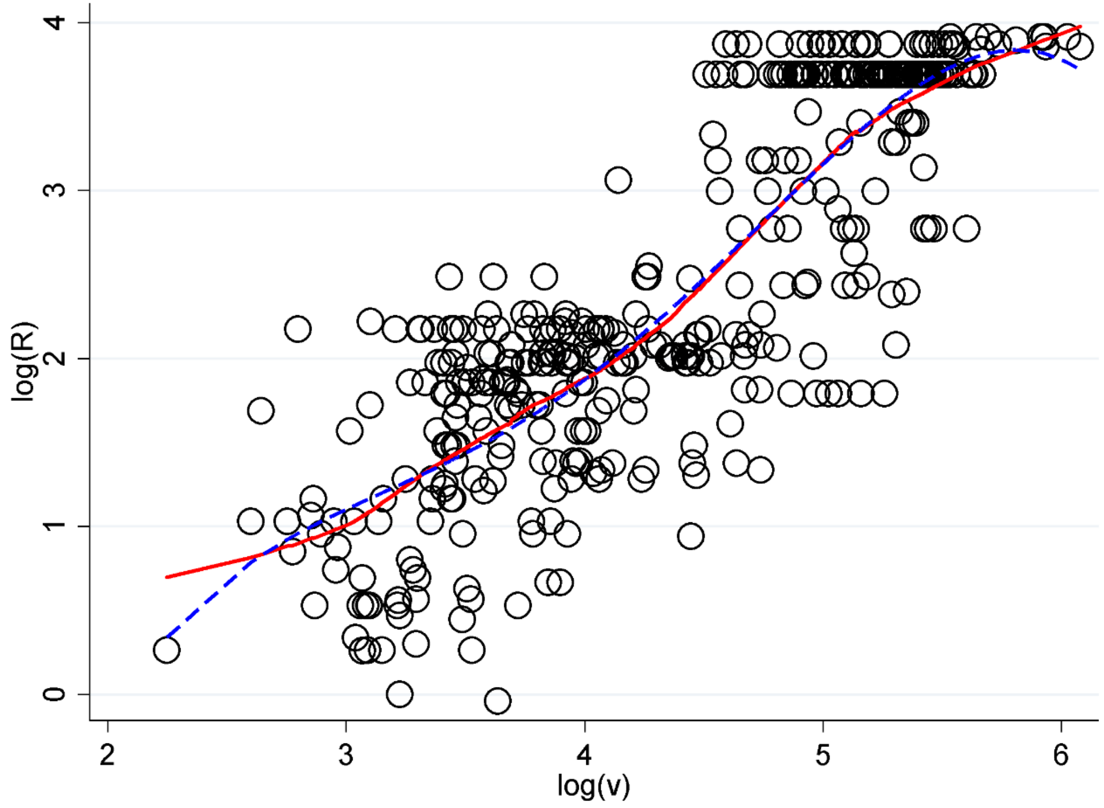


Residential

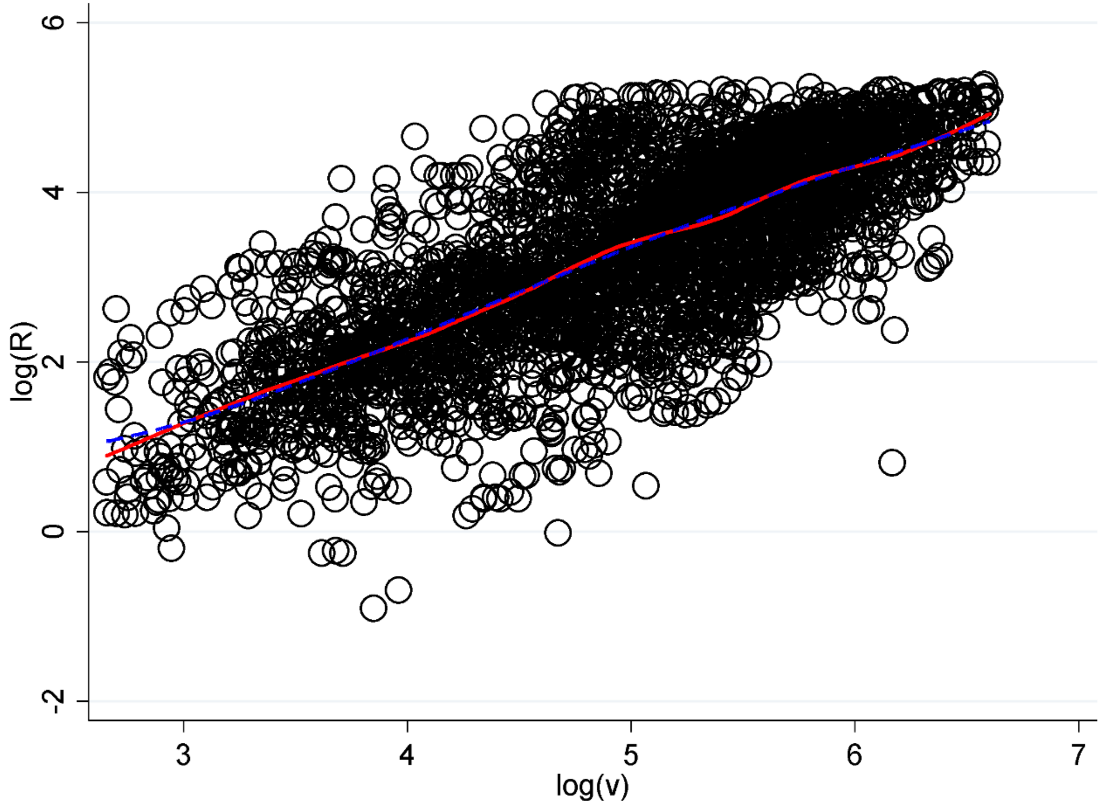


Commercial

Estimates for Chicago

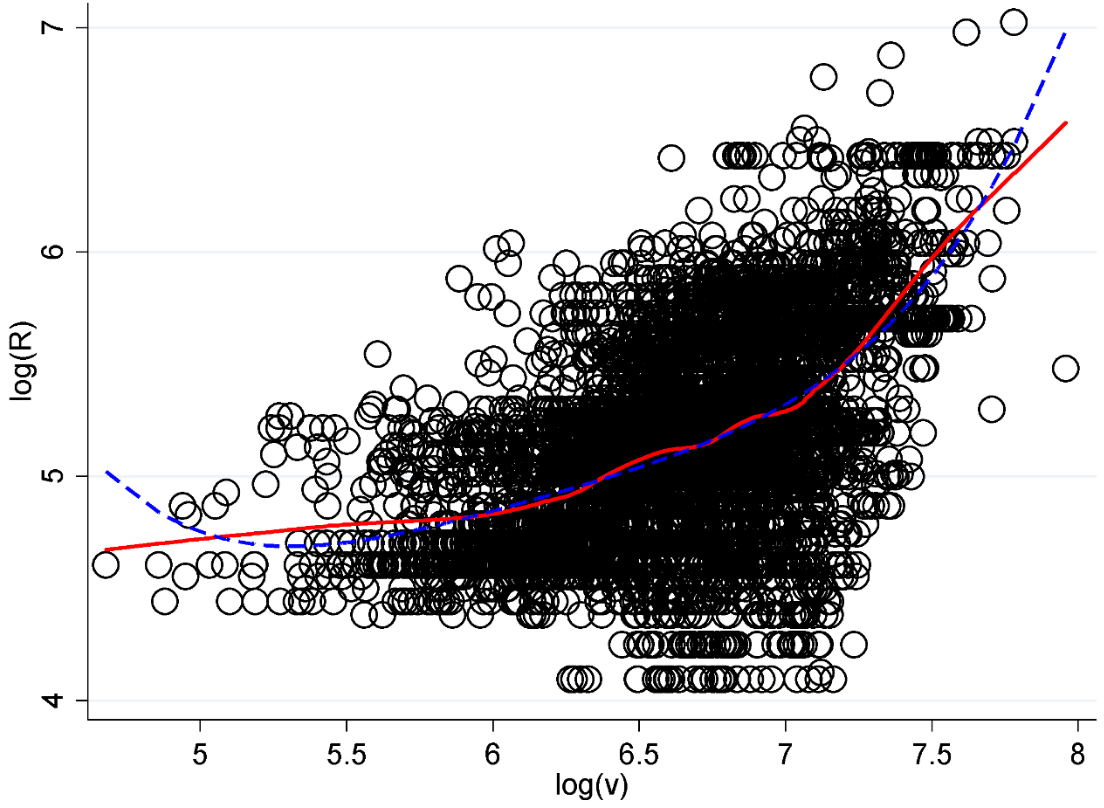


Olcott's

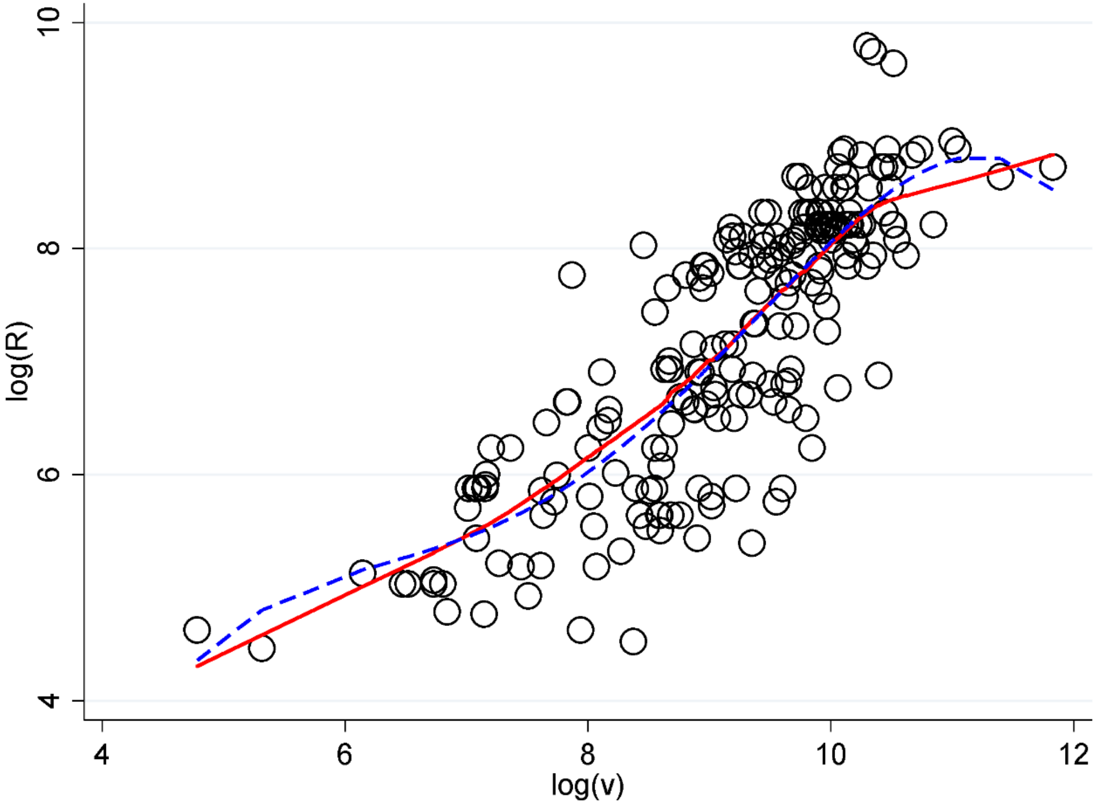


Vacant Land Sales

Estimates for Berlin



Residential



Commercial

Estimated Elasticities

		Classic approach		EGS Approach	
Data set	Obs.	OLS	IV	LWR	Log LWR
Allegheny County Residential	6362	0.95^{***}	1.36^{***}	1.13^{***}	1.13^{***}
Allegheny County Commercial	992	0.93^{***}	1.29^{***}	1.44^{***}	1.44^{***}
Chicago Residential, Olcott's	414	0.60^{***}	0.85^{***}	0.95^{***}	0.91^{***}
Chicago Residential, Vacant Land	3576	0.43^{***}	0.88^{***}	1.02^{***}	0.97^{***}
Berlin Residential	5466	0.286^{***}	1.186^{***}	1.731^{***}	1.834^{***}
Berlin Commercial	273	0.732^{***}	0.903^{***}	1.222^{***}	1.202^{***}
Mean		0.65	1.08	1.25	1.25