Ambiguity in Macroeconomics and Finance

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Why study ambiguity in macro?

- Rational expectations (RE) as the standard paradigm
  1. agents are SEU maximizers
  2. subjective and objective probability distributions (i.e. models) coincide.
- Extend RE to acknowledge concern for model misspecification (Hansen and Sargent):
  - econometricians *and* agents (including policy makers) face uncertainty over the model
  - positive: explain the macro (and micro) data 'better'
- Recent literature on ambiguity aversion that shows promise
  - new insights
  - tractable quantitative models that are parsimonious and fit well
- Today: themes from chapter on Ambiguity, with Martin Schneider, in Handbook on Economic Expectations

Themes: (1) Choice under ambiguity

- Common denominator: higher uncertainty $\rightarrow$ *more cautious* behavior
- Main **qualitative** differences:

	expected utility requires	**ambiguity aversion allows for**
welfare cost of uncertainty	2nd order	1st order (*as if* lower mean)
uncertainty affects actions	curvature	linear utility
precautionary savings	prudence > 0	IES $< \infty$
take on small uncertain project	almost always (local risk neutrality)	inaction / inertia

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(2) Building DSGE models with ambiguity

- Common denominator: risk/ambiguity as a state variable alter model dynamics
  - high ambiguity = low confidence in probability assessments

- Important **methodological** differences

	expected utility	**ambiguity aversion**
computation	nonlinear methods	steady state + linear dynamics
observable proxy for uncertainty	conditional volatility (statistical or subjective)	conditional volatility expert forecast disagreement new direct survey evidence
model consistency	rational expectations	belief bounds

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1. Choice under ambiguity
   - preferences; first order welfare effects
   - applications: precautionary savings, portfolio inertia
2. Building DSGE models with ambiguity
   - dynamics: preferences, time-varying confidence
   - discipline: model consistency, surveys
   - equilibrium
3. A specific application: HANK model with ambiguity
   - ambiguity about aggregate TFP
   - solution method
   - main findings
4. Conclusion
   - brief review of further applications and open areas

Key objects: preferences over consumption plans

- Two dates, one good; at date 2 a state $\omega \in \Omega$ is realized; the agent chooses a *plan* $(c, \tilde{c})$
- Models we consider are represented by

$$u(c) + \beta u(CE(\tilde{c})) \tag{1}$$

  - u ranks certain plans (curvature controls intertemporal substitution)
  - CE is the certainty equivalent of date 2 consumption
  - *CE's curvature* says whether the agent is averse to risk or to ambiguity
- **Risk aversion**: a single subjective belief P

$$v(CE(\tilde{c})) = E^P[v(\tilde{c})] \tag{2}$$

  - curvature in v is risk aversion
  - $u \neq v$ allowed (Epstein-Zin), $u = v$ gives time-separable expected utility

Multiple priors

- **Multiple priors** (Gilboa & Schmeidler, 1989): a *set* $\mathcal{P}$ of beliefs

$$v\left(CE\left(\tilde{c}\right)\right) = \min_{P \in \mathcal{P}} E^P\left[v\left(\tilde{c}\right)\right] \tag{3}$$

  - act *as if* using the most pessimistic $P^* \in \mathcal{P}$ a singleton returns (2)
  - size of $\mathcal{P}$ = lack of confidence
  - P^* varies *endogenously* with the plan $\rightarrow$ Ellsberg behavior
- Behavior: isn't the worst-case scenario paranoid?
  - extent of caution in behavior governed by width of the set $\mathcal{P}$
  - eg. $\mathcal{P}$ can be (very) narrow $\rightarrow$ almost SEU & less cautious than with high risk aversion
- **Other models**: multiplier preferences, smooth ambiguity – also use belief sets and endogenous pessimism, but make *CE smooth* in the plan

Risk vs. ambiguity: uncertainty without curvature

- Both certainty equivalents, (2) and (3), penalize plans that *vary across states*
  - standard intuition: a lower CE makes agents unhappy, and may act precautionarily
- But the models differ in what *an increase in uncertainty* means
  - risk: a mean preserving spread in the *distribution* of date 2 consumption
  - ambiguity: an increase in the *set* of distributions used to evaluate it
- Risk aversion: the utility cost of uncertainty comes from curvature in v
  - if v is linear, a mean preserving spread does not affect utility – CE depends only on $E^P[\tilde{c}]$
  - eg. search markets, or contracting within firms, use linear $u = v$
- Multiple priors: uncertainty aversion survives with *linear* $u = v$

$$c + \beta \min_{P \in \mathcal{P}} E^P[\tilde{c}] \tag{4}$$

  - a larger set lowers utility if it lowers the worst-case mean

Welfare costs of uncertainty: second vs. first order

- **Risk aversion: second order**
  - start from a certain plan $\tilde{c} = \bar{c}$ and add a small mean preserving spread $\alpha\tilde{z}$, with $E^P\tilde{z} = 0$
  - concave v: welfare declines for any α, but the loss scales with α^2
    - agents tolerate high uncertainty in return for a small gain in the mean
- **Multiple priors: first order**
  - linear utility (4), add $\alpha\tilde{z}$ with an *interval of means bracketing zero* (other moments identical)

$$\min_{P\in\mathcal{P}} E^P\tilde{z} < 0 < \max_{P\in\mathcal{P}} E^P\tilde{z},$$

  - utility declines for any α: the worst-case mean *endogenously* adjusts so its sign is opposite to that of α
  - key: welfare loss is *linear* in $\alpha \Rightarrow$ mean and uncertainty can have the same magnitude order

Timing of the resolution of uncertainty

- Common practice under risk aversion: curvature in v much larger than in u
  - prominent in macro and finance, to address asset pricing puzzles under rational expectations
- But it also implies a strict preference for *early* resolution of uncertainty
  - pay to receive info *even if agents cannot adjust their plans* in response (Epstein et al., 2014)
  - standard expected utility agents, $u = v$, do not behave this way
- Multiple priors breaks the link between *aversion to* uncertainty and *timing of* its resolution
  - with $u = v$, agents are *indifferent* to timing, just like expected utility maximizers
  - eg. unit IES with a large set $\mathcal{P}$: requires a lot of compensation for uncertainty, yet would not pay up front for information unless it can be used in planning
  - if compensation for timing *is* desirable to match the data, augment with $v \neq u$

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Precautionary savings: setup

- Two-period consumption-savings problem
  - date 1 wealth w, uncertain date 2 labor income $\tilde{y}$
  - savings $s \geq 0$ earn a certain gross return R

$$c = w - s; \qquad \tilde{c} = Rs + \tilde{y}$$

  - abstract from timing effects: $u = v$
- For *both* risk aversion and multiple priors, interior optimum solves the same Euler equation

$$u'(w - s) = \beta R \, E^{P^*}\left[u'(Rs + \tilde{y})\right] \tag{5}$$

Precautionary savings: risk vs. ambiguity

- Risk aversion: textbook result – income uncertainty raises savings *iff* prudence, $u''' > 0$
  - the tractable case of *quadratic utility rules it out*
  - a linear approximation to (5) reflects at most $u'' \rightarrow$ cannot be used to study it
  - $\Rightarrow$ solving GE models by linearization abstracts from precautionary savings
- Multiple priors: precautionary savings requires only *curvature* in u
  - let $\tilde{y} = \bar{y} + \alpha \tilde{z}$, $\alpha > 0$, where the interval of means for $\tilde{z}$ brackets zero
  - worst case P^*: the belief with the lowest mean for $\tilde{z}$ (minimizes EU regardless of s and α)
  - implicit function theorem applied to (5):

$$\left.\frac{ds}{d\alpha}\right|_{\alpha=0} = \frac{1}{\Delta} R u'' \left(Rs + \bar{y}\right) E^{P^*} \left[\tilde{z}\right] > 0$$

  - holds as long as the agent is not willing to perfectly substitute consumption over time

Two principles this example illustrates

1. Ambiguity aversion: observationally equivalent to **pessimism**, *for a given choice problem*
   - solve in two steps: find the worst-case belief up front, then solve with familiar tools
   - convenient for computation, but it does *not* extend beyond that problem
   - example: a transfer program paying $-2\alpha\tilde{z}$ at date 2
     - ★ subsidizes bad states and taxes good ones, enough to reverse their roles
     - ★ fixed-prior pessimist: *better off* – more income in the states he finds likely
     - ★ ambiguity averse agent: utility *unchanged* – same family of distributions, so the worst-case mean shifts endogenously to the now-bad states
2. **Curvature in the objective** governs the response to uncertainty
   - the more reluctant to substitute consumption over time, the more precautionary saving
   - with linear u: the agent still worries, utility is lower, but does nothing about it
   - in dynamic models, anything that makes uneven plans costly (e.g. adjustment costs) strengthens precautionary behavior

Portfolio inertia

- Two assets, abstract from the savings margin
  - mean-variance investor over end-of-period wealth
  - excess return over the risk-free rate $\tilde{R} = \bar{R} + \tilde{z}$
  - belief set: the interval of means for $\tilde{z}$ brackets zero, all other moments the same
  - θ: zero-cost portfolio, short the bond and long the uncertain asset

$$\max_{\theta} \min_{P \in \mathcal{P}} \; E^P \left[\tilde{R}\right] \theta - \frac{1}{2} \gamma \, var^P \left[\tilde{R}\right] \theta^2$$

- Without ambiguity: $\theta = E^P[\tilde{R}]/\gamma \, var^P[\tilde{R}]$
  - participate whenever the subjective expected excess return is not *exactly* zero
  - any small compensation for risk taking suffices – the welfare loss is second order
  - any change in the expected excess return alters the optimal portfolio

Portfolio inertia: the mechanism

- With ambiguity, non-participation $\theta = 0$ is optimal when (Dow & Werlang, 1992):

$$\bar{R} + \min_{P \in \mathcal{P}} E^P \tilde{z} < 0 < \bar{R} + \max_{P \in \mathcal{P}} E^P \tilde{z}$$

  - contemplate going *long*: the worst-case expected excess return is below zero
  - contemplate going *short*: it is above zero

  $\Rightarrow$ the objective is negative either way, so $\theta = 0$ is strictly better

- Key: the endogenous switch to a pessimistic belief *that depends on the action*
- Outside the region: if $\bar{R}$ is sufficiently above (below) zero, the solution is the same as under risk – but evaluated at the lowest (highest) mean of $\tilde{z}$

$\Rightarrow$ as a function of $\bar{R}$, the optimal portfolio has an **inaction region**

  - out of the market, and unresponsive to changes in $\bar{R}$
  - no fixed or transaction costs needed
  - confirmed in laboratory asset markets (Bossaerts et al., 2010; Asparouhova et al., 2015)

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Ambiguity aversion & preferences

- S = state space
  - one element $s \in S$ realized every period; histories $s^t \in S^t$
- Consumption streams $C = (C_t(s^t))$
- Recursive multiple-priors utility (Epstein and Schneider (2003))

$$U_t\left(C; s^t\right) = u\left(C_t\left(s^t\right)\right) + \beta \min_{p \in \mathcal{P}_t(s^t)} E^p\left[U_{t+1}\left(C; s^{t+1}\right)\right],$$

- Primitives:
  - felicity u and discount factor β
  - one-step-ahead belief sets $\mathcal{P}_t\left(s^t\right)$ – size captures (lack of) confidence

Ambiguity in linear models

- DSGE model: s^t = history of innovations to exogenous shocks
- Representation of one-step-ahead belief set $\mathcal{P}_t$ for shock x_i :

$$x_{t+1,i} = \rho_i x_{t,i} + \sigma_{t,i}\varepsilon_{t+1,i} + \mu_{t,i}$$
$$\mu_{t,i} \in [-a_{t,i}, a_{t,i}]$$

- Stochastic process $a_{t,i}$ regulates size of $\mathcal{P}_t(s^t)$
  - larger $a_{t,i}$ = larger set = less confidence about shock $x_{t+1,i}$
  - min operator selects worst case mean, e.g. $-a_{t,i}$
  - if ambiguity $a_{t,i}$ increases, agent acts "as if" bad news about $x_{t+1,i}$

Evolution of confidence

Describe ambiguity by

$$a_t = \eta_t \widehat{\sigma}_t$$

1. Intangible information: η_t
   - worrisome news about future
2. Stochastic volatility: $\widehat{\sigma}_t = \sigma_t$
   - turbulence in observed data lowers confidence
3. Learning: perceived volatility $\widehat{\sigma}_t = \sigma(x_{t+1}|I_t)$
   - estimation uncertainty lowers confidence

Linearity follows if $\mathcal{P}_t$ is relative entropy ball around $\mu_t = 0$:

$$\frac{\mu_t^2}{\widehat{\sigma}_t^2} \leq \eta_t^2$$

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Discipline

1. Model consistency criteria: discipline reasonable sets of beliefs, relax rational-expectations
   - how much doubt is *statistically* defensible?
   1. Detection error probability (Hansen and Sargent)
   2. Bounds in a non-stationary worlds (Ilut and Schneider)
2. Empirical quantification

Model consistency discipline (1): detection error probability

- Compare two stationary models: the 'true' and the 'distorted' one
  - the distorted model is the *endogenous* worst case – it comes out of the decision problem
  - hand an econometrician a sample of length T and a likelihood ratio test
  - two ways to be wrong: pick distorted when true is right, and vice versa
  - detection error probability = average of the two error rates, equal priors
  - computed by simulation; invert to choose robustness, e.g. 10%
- How to use it
  - near 1/2: models nearly indistinguishable $\rightarrow$ fear of the worst case is reasonable
  - near 0: worst case is a straw man the data would reject $\rightarrow$ too much robustness
  - with increasing amount of data, will be able to distinguish them: doubt is harder to justify

Model consistency discipline (2): Non-stationarity concerns

- Recall: one-step-ahead belief set $\mathcal{P}_t$

$$x_{t+1,i} = \rho_i x_{t,i} + \sigma_i \varepsilon_{t+1,i} + \mu_{t,i}; \qquad \mu_{t,i} \in [-a_{t,i}, a_{t,i}]$$

- True data generating process
  - deterministic sequence $\mu^*_{t,i}$ with moments converging to $i.i.\mathcal{N}\left(0, \sigma^2_{\mu,i}\right)$
  - neither agents nor econometrician can identify true $\mu^*_{t,i}$: only observe compound innovation

$$\Delta x_{t,i} \equiv x_{t,i} - \rho_i x_{t-1,i} = \sigma_i \varepsilon_{t,i} + \mu_{t-1,i}$$

  - impossible to distinguish various models (i.e. sequence $\mu^*_{t,i}$) with any amount of data
  - agent does not resolve uncertainty probabilistically, but treats it as ambiguity
- Upper bound on ambiguity: even worst-case forecasts are 'good enough' in the long run
  - do not entertain forecasts outside a 95% confidence interval, centered around the long-run conditional mean of $\Delta x_{t+1,i}$ given its observed variation
  - implies $a_t < 2\sigma_x$, where σ_x is observed long run variance of measured innovation $\Delta x_{t+1,i}$

Beliefs vs. data

Empirical discipline: what surveys can do

- So far: bounds imposed *a priori* – can we measure ambiguity instead?
  - large scale surveys of households and firms
  - (experimental evidence: survey by Trautmann & Van De Kuilen, 2015)
- Three takeaways
  1. ambiguity aversion is **widespread**: households and decision makers in firms alike
  2. variation in ambiguity is **large**: across agents, and over time for a given agent
     partly accounted for by *information* – but not by *sophistication*
  3. measured ambiguity relates in **sensible ways to observed choice**

$\Rightarrow$ evidence is consistent with rational behavior under ambiguity, so surveys are a promising source of discipline for belief sets

Methodology: how to read an imprecise probability

- Elicit stated **"imprecise probabilities"**
  - ask for the likelihood of an event, but allow a *range* as the answer
  - roots in psychology (Wallsten et al., 1983); into economics with Manski and Molinari (2010)
- Decision theory does *not* map belief sets into survey answers
  - belief sets are devices to represent *preferences* – agents act *as if*
  - in particular, theory does **not** predict that ambiguity averse agents forecast pessimistically
    - ★ just as expected utility does not predict that risk averse agents do
- Two statements survive
  1. a Bayesian should answer with a *single* probability

     → frequency of imprecise answers measures prevalence of *non-Bayesian* perception of uncertainty
  2. more ambiguity averse agents behave as if their belief sets are larger

     → *add* the assumption that they also state wider ranges: can then rank ambiguity across agents, or for one agent over time

New evidence on firms

- Bachmann et al. (2020): imprecise probabilities about future *sales growth*, from leading executives in German manufacturing (ifo Business Survey)
  - quarterly panel over four years
- Ambiguity is pervasive even for a routine, short-term variable
  - 25% of firms answer with imprecise probabilities in a given quarter
  - 70% do so at least once
- It moves with familiar measures of uncertainty
  - time variation systematically related to *credit spreads*
  - share of ambiguous responses spikes in the 2015 Greek default crisis – especially for exporters
- It is *not* a sophistication story
  - unrelated to the use of statistical analysis inside the firm
  - forecast mistakes similar for precise and imprecise responses

Surveys and quantified models

1. **Jointly design** the survey and the structural model
   - tie survey answers tightly to their model counterparts
   - Giustinelli & Pavoni (2017): learning under ambiguity by Italian high school students choosing career tracks; panel on beliefs and choices
   - elicited imprecise probabilities are moments that discipline the *evolution* of uncertainty
2. **Dispersion of expert forecasts in survey** as a proxy for ambiguity
   - Ilut & Schneider (2014), SPF: ambiguity averse household samples experts' opinions; stronger disagreement among experts $\rightarrow$ lower confidence
   - beliefs $=$ interval for the one-quarter-ahead conditional mean of TFP, monotone in the interdecile range of expert forecasts over endogenous vars
   - dispersion's volatility and comovement discipline the scope for uncertainty to matter

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General framework: Rep. agent & Markov uncertainty

- Notation (as before s = exogenous state)
  - X = endogenous states (eg. capital); A = agent actions (eg. consumption, investment); Y = other endogenous variables (eg. prices)
- Recursive equilibrium
  - Functions for actions A, other endog vars Y, value V s.t., for all (X, s):

$$V(X,s) = \max_{A \in B(Y,X,s)} \left\{ u\left(c\left(A\right)\right) + \beta \min_{p \in \mathcal{P}(s)} E^{p}\left[V(X', s')\right] \right\}$$
$$s.t.\ X' = T(X, A, Y, s, s')$$

  - endog var determination: $G\left(A, Y, X, s\right) = 0$
  - true exogenous Markov state process $p^{*}\left(s\right) \in \mathcal{P}\left(s\right)$
- Analysis in 2 steps
  1. find recursive equilibrium
  2. characterize variables under "true" state process

Characterizing equilibrium: a guess-and-verify approach

- basic idea
  1. guess the worst case belief p^0
  2. find recursive equilibrium under expected utility & belief p^0
  3. compute value function under worst case belief, say V^0
  4. verify that the guess p^0 indeed achieves the minimum
- "essentially linear" economies & productivity shocks
  - environment T, B, G s.t. 1st order approx. ok *under expected utility*
  - ambiguity is about mean of innovations to s
- simplification in essentially linear case
  - in step 1, guess that worst case mean is linear in state variables
  - step 2 uses loglinear approximation around "distorted" steady state (sets risk to zero, but retains worst case mean)
  - step 4 then checks monotonicity of value function

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HANK with aggregate uncertainty as ambiguity

- "HANK's Response to Aggregate Uncertainty in an Estimated Business Cycle Model" (Ilut, Luetticke and Schneider)
- Large interest in quantitative models with heterogeneity and nominal rigidities
  - frictions in financial markets $\rightarrow$ agents' response to changes in uncertainty matters
- But typically restrict attention to effects of idiosyncratic uncertainty
  - HANK models are typically solved at first-order with respect to aggregate shocks
  - with expected utility, uncertainty has only second-order effects on utility and choice

  $\rightarrow$ ignore role of aggregate uncertainty for precautionary savings, premia, firm decisions
- Disconnect between HANK and macro-finance literature:
  - time-varying aggregate uncertainty is key for business cycles and asset prices

Ambiguity about Aggregate TFP

- Parameterize one-step ahead belief sets by the mean of TFP innovations

$$\begin{aligned} \log Z_{t+1} &= \rho_z \log Z_t + \mu_t + \epsilon^Z_{t+1}; \quad \epsilon^Z \sim i.i.dN(0,\sigma_z) \\ \mu_t &\in [-a_t, a_t] \end{aligned}$$

- Higher a_t $\rightarrow$ larger belief set $\rightarrow$ more ambiguity about TFP in $t+1$
- Stochastic process for a_t:

$$a_t - \bar{a} = \rho_a(a_{t-1} - \bar{a}) + \epsilon^a_t$$

  - long run mean $\bar{a} > 0$, persistence $0 \leq \rho_a < 1$, and $\epsilon^a_t \sim i.i.d\ N(0,\sigma_a)$
- This model: worst case belief is always low mean TFP

$$\mu^*_t = -a_t$$

  - pessimism in mean = first order effects of uncertainty!

Ambiguity and Decision Rules

1. Objective of the firm
   - Ambiguity is about the mean: *as if* risk-neutral owner with $\mu_t^* = -a_t$
   - all agents share that common belief = objective of the firm well defined
2. Correlated wedges: precautionary motive in *all* intertemporal decisions
   - households save & choose portfolios *as if* future expected wages & returns are low
     $\rightarrow$ interest rate reflects benefit of safety
     $\rightarrow$ capital premium reflects compensation for uncertainty
   - firms invest & set prices *as if* future expected cost is high
3. Given equilibrium law of motion, characterize path of variables under true DGP

$$\log Z_t = \rho_z \log Z_{t-1} + \epsilon_t^Z$$

Solution method

Reiter, 2009: solve HH problem globally + approximate aggregate dynamics by perturbation

Perturbation: main steps

1. Compute deterministic steady-state $(Z^*, \mathbf{Y}^*)$ - at worst-case TFP, $Z^* = \bar{Z} \exp\left(\frac{-\bar{a}}{1-\rho_z}\right)$
2. Linear approximation of model dynamics, with ambiguity, around $(Z^*, \mathbf{Y}^*)$
3. Simulate under true DGP $\rightarrow$ ergodic distribution of model with aggregate uncertainty
   - change of measure in TFP law of motion: $\log Z_t = E^*_{t-1} \log Z_t + a_{t-1} + \epsilon^Z_t$
   - ergodic steady state $(\bar{Z}, \bar{\mathbf{Y}}) \neq (Z^*, \mathbf{Y}^*)$

Estimation: very tractable due to linearization

Main findings: HANK's response to aggregate uncertainty

1. Steady state: aggregate uncertainty helps fit key moments with less illiquidity
   - Capital premium: compensation for uncertainty (3.2% premium) & illiquidity (2.3%)
   - Generates HtM households with less portfolio frictions
   - Frictions *amplify* uncertainty: 0.11% uncertainty premium in counterfactual RANK
2. Dynamics: HANK frictions *amplify* the business cycle effects of aggregate uncertainty
   - Ambiguity about TFP $\rightarrow$ comovement of key macro aggregates & asset prices (jointly explain $>$ 70% of cyclical variation)
   - Strong substitution distinguishes aggregate from idiosyncratic uncertainty shocks
   - Fit investment and excess return on capital much better than a RANK version

Mechanism: marginal investor more exposed to the uncertain capital returns than rep agent

Further applications: what does ambiguity add?

- Dynamics of **macro variables & asset prices** over the business cycle
  - eg. low ambiguity/high confidence $\rightarrow$ economic boom with
    - low premia, high real rate, high Consumption, Investment, Hours
  - time-varying ambiguity: shocks or endogenous learning
- **Micro-to-macro**: behavior of heterogeneous investors & firms
  - *heterogeneous* perceptions of uncertainty
    - eg. lender (borrower) worries about low (high) real interest rate
  - inertia: mechanism for *price rigidity*
    - when increase (decrease) price worry that demand is elastic (inelastic)
  - *asymmetric decision rules*: respond more to bad than to good news
    - endogenous neg skew & countercyclical vol in X-section & aggregate
- **Optimal policy**
  - not confident in model parameters $\rightarrow$ policy inertia or aggressiveness
  - large welfare cost of (policy) uncertainty

Looking ahead

1. Models where aggregate uncertainty matters: ambiguity helps vs. risk
   - capture features of uncertainty that are plausible: eg. trade policy, climate change
   - tractability in large state models: household and firm heterogeneity
2. Optimal policy, especially where there is insufficient data (eg. AI, climate policy)
3. Joint dynamics of uncertainty and observables, with feedbacks
   - existing work on how beliefs shape data
   - need more work on how data shapes beliefs
   - theory: models of learning
   - empirics: surveys data on how beliefs evolve

Beliefs versus data

- True DGP for shock x

$$x_{t+1} = \rho x_t + \tilde{\sigma}_t \varepsilon_{t+1} + \mu_t^*$$

  - deterministic sequence $\{\mu_t^*\}$ unknown
    empirical moments same as iid normal process with mean zero & variance σ_μ^2
  - cannot identify $\mu_t^*, \tilde{\sigma}_t$ without further assumptions

- Econometrician
  - resolve uncertainty probabilistically by assuming stationarity
  - represent uncertainty as risk

$$x_{t+1} = \rho x_t + \widehat{\sigma}_t \varepsilon_{t+1}$$

    where $\widehat{\sigma}_t^2 = \tilde{\sigma}_t^2 + \sigma_\mu^2$

- Agents
  - consider nonstationary models given by different μ_t^*s
  - treat μ_t^* as ambiguous
  - respond to uncertainty as if minimizing over $[-a_t, a_t]$

Back

Magnitudes of sets

'Consistency' criterion

- Constrain a_t to lie in a maximal interval $[-a^{\max}, a^{\max}]$
  - require that "boundary" beliefs $\pm a_{\max}$ imply "good enough" forecasts:
  - $a_{\max}$ is best forecasting rule at date t if true mean $\mu_t^* > a_{\max}$
- $\Rightarrow$ Upper bound on $a^{\max}$:
  - for any true sequence μ_t^*, "most extreme" forecasts $\pm a^{\max}$ are best forecast at least $x\%$ of the time
  - for example, $x = 5\%$ implies $a_{\max} = 2\sigma$

Back