Robust Contract Design

Alessandro Pavan

Contract/Mechanism Design

- Contract/mechanism design
  - central to many economic problems

    - procurement

    - regulation

    - team design

    ...

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- Difficulties
  - **asymmetric information**
  - **uncertainty**

    - agents' higher-order information (Ben's talk)

    - other primitives (e.g., type distribution, demand, technology)

Different Approaches

- Bayesian/SEU
- Worst-case optimality
- Bayesian design with conservative payoff guarantee constraints
- Lexicographic max-min
- Competitive ratio
- Min-max regret

Literature

- **Bayesian (subjective expected utility) design**
  - procurement: Baron-Myerson (1982)...
- **Worst-case optimality**
  - ...Garrett (2014)...
- **Bayesian design with conservative payoff guarantees**
  - Mishra-Patil-Pavan (2026)
- **Lexicographic max-min**
  - Kambhampati (2025)
- **Competitive ratio**
  - Bergemann-Heumann-Morris (2025)
- **Min-max regret**
  - Guo-Shmaya (2024)
- **Also related:**
  - **Robust control, model mis-specification**
    - Bergemann-Morris (2005), Bergemann-Schlag (2008), Lopomo-Rigotti-Shannon (2021), Cerreia-Vioglio-Hansen-Maccheroni-Marinacci (2025)
  - **Un-dominated design**
    - Mishra-Patil (2025), Borgers-Li-Wang (2025)
  - **Robust information design**
    - Dworczak-Pavan (2023), Cheng-Klibanoff-Mukerji-Renou (2024)

Plan

1. Introduction
2. **Procurement**
3. Bayesian design
4. Worst-case optimality
5. Bayesian design with conservative payoff guarantees
6. Lexicographic max-min
7. Competitive ratio
8. Min-max regret
9. Conclusions

Procurement Environment

Environment

- **Players**
  - Designer (buyer)
  - Agent (supplier)
- **Contracting variables**
  - output $q \in [0, \bar{q}]$ (intensity of trade)
  - transfer t

Environment

- **Payoffs**
  - Value of q to designer:
    $$V(\mathrm{q}) = \int_0^{\mathrm{q}} P(s)ds$$
    P is inverse demand, $D = P^{-1}$ is direct demand
  - Seller's cost $\theta\mathrm{q}$
  - Players' outside option payoff: 0
  - Designer payoff:
    $$V(\mathrm{q}) - \mathrm{t}$$
  - Seller's payoff:
    $$\mathrm{t} - \theta\mathrm{q}$$
- **Asymmetric information**
  - θ: seller's private information

Plan

Bayesian Design

Bayesian approach

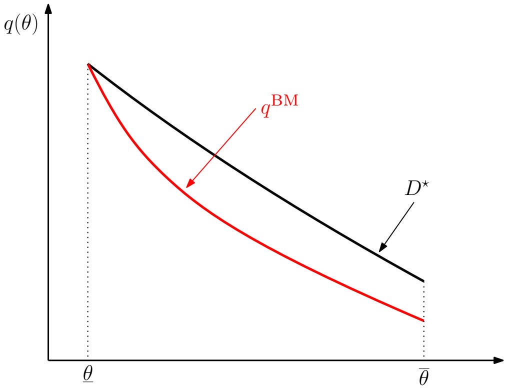
- **Designer has model** $(V^\star, F^\star)$
  - Value function: $V^\star(\mathrm{q}) = \int_0^{\mathrm{q}} P^\star(s)ds$
  - Cost technology: θ drawn from abs. cont $F^\star$ with $f^\star(\theta) > 0$ over $\Theta = [\underline{\theta}, \overline{\theta}]$ and "**virtual cost**"
    $$z^\star(\theta) := \theta + \frac{F^\star(\theta)}{f^\star(\theta)}$$
    cont. and increasing (**regularity**)
- **(Direct) procurement mechanism** $M = (q, t)$
  - quantity schedule $q : \Theta \to \mathbb{R}_+$
  - transfer schedule $t : \Theta \to \mathbb{R}$
  - Rent to type θ:
    $$u(\theta) := t(\theta) - \theta q(\theta)$$
- $\mathcal{M}$: set of IC and IR mechanisms
- Given $M = (q, u) \in \mathcal{M}$, **expected welfare** under $(V^\star, F^\star)$

$$W(M; V^\star, F^\star) := \int [V^\star(q(\theta)) - \theta q(\theta) - u(\theta)]\, F^\star(\mathrm{d}\theta)$$

SEU/Bayesian Benchmark: Baron Myerson

Proposition

SEU/Bayesian optimum: $q^{BM}(\theta) := D^\star(z^\star(\theta))$, where $z^\star(\theta) := \theta + \frac{F^\star(\theta)}{f^\star(\theta)}$ is "virtual cost".



(BM-Proposition)

Plan

Worst Case Optimality

Worst-Case Optimality

- Designer has **admissible set (of models):** $\mathcal{A} = \mathcal{V} \times \mathcal{F}$
  - $\mathcal{V}$: set of possible (gross) value fns
  - each $V \in \mathcal{V}$ abs continuous, increasing, concave
    - $\mathcal{P}$: set of corresponding *inverse demand functions*, with

$$V(q) = \int_0^q P(s)ds \qquad \forall q$$

    - $\mathcal{D}$: set of corresponding *direct demand functions*, $D = P^{-1}$
    - Lowest (inverse) demand function: $\underline{P}$

      any $q \geq 0$ and $P \in \mathcal{P}$

$$P(q) \geq \underline{P}(q)$$

      $\underline{P}$: strictly decreasing, continuous, and s.t.

$$\lim_{q \to 0^+} \underline{P}(q) > \overline{\theta}$$

  - $\mathcal{F} = \text{CDF}(\Theta)$: all cdfs with support contained in $\Theta = [\underline{\theta}, \overline{\theta}]$

    (more general $\mathcal{F}$, w. lowest element $\underline{F}$ can be considered)

Environment

- $\mathcal{M}$: set of IC and IR mechanisms
- Designer's welfare under $M \in \mathcal{M}$, given (V, F):

$$W(M; V, F) := \int [V(q(\theta)) - \theta q(\theta) - u(\theta)] \, F(\mathrm{d}\theta)$$

Definition

Welfare guarantee for $M \in \mathcal{M}$,

$$G(M) := \inf_{(V,F) \in \mathcal{A}} W(M; V, F).$$

Environment

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Definition

Worst case optimality

$$\max_{M \in \mathcal{M}} G(M)$$

Worst-case Optimality

- Let

$$\mathrm{q}_\ell := \arg\max_{\mathrm{q}} \left\{ \underline{V}(\mathrm{q}) - \bar{\theta}\mathrm{q} \right\} = \underline{D}(\bar{\theta})$$

be efficient output when $V = \underline{V}$ and $\theta = \bar{\theta}$ and

$$G^* := \underline{V}(\mathrm{q}_\ell) - \bar{\theta}\mathrm{q}_\ell$$

Lemma

For any IC and IR mechanism $M = (q, u) \in \mathcal{M}$

$$G(M) = \inf_{\theta \in \Theta} \left\{ \underline{V}(q(\theta)) - \theta q(\theta) - \int_\theta^{\bar{\theta}} q(y) dy - u(\bar{\theta}) \right\}.$$

Furthermore,

$$G(M) \leq G^*$$

Worst-case Optimality

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Furthermore,

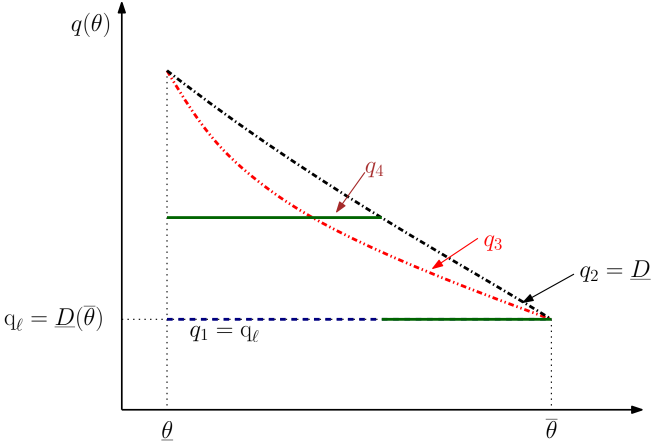
$$G(M) \leq G^*$$

- Worst case always under lowest demand **but not necessarily highest cost**

(Max-guarantee-proof)

Worst-case Optimality

- "Flat" mechanism $M_1 = (q_1, t_1)$ with $q_1(\theta) = \mathrm{q}_\ell$ all θ, delivers maximal guarantee G^*
- ...so does $M_2 = (q_2, t_2)$ with $q_2(\theta) = \underline{D}(\theta)$ all θ
- ...but also mechanisms $M_3 = (q_3, t_3)$ and $M_4 = (q_4, t_4)$...
- ...continuum of worst-case optimal mechanisms



Worst-case Optimality: Full Characterization

Lemma

$M = (q, u)$ is worst-case optimal iff

1. *q non-increasing,*
2. *$u(\theta) = \int_\theta^{\overline{\theta}} q(y)dy$ all θ*
3. *for all θ,*

$$\underline{W}(\theta, q) := \underline{V}(q(\theta)) - \theta q(\theta) - \int_\theta^{\overline{\theta}} q(y)dy \geq G^*$$

WCO: Majorizations

Lemma

Take any non-increasing q s.t. $q(\overline{\theta}) = \mathrm{q}_\ell$. *Following are equivalent*

$$\underline{W}(\theta, q) := \underline{V}(q(\theta)) - \theta q(\theta) - \int_\theta^{\overline{\theta}} q(y)dy \geq G^* \qquad \forall \theta \in \Theta$$

$$\int\limits_\theta^{\overline{\theta}} q(y)dy \leq \int\limits_\theta^{\overline{\theta}} \underline{D}(y)dy - \underbrace{\int\limits_\theta^{\underline{P}(q(\theta))} \left(\underline{D}(y) - q(\theta)\right) dy}_{DWL(\theta, q(\theta)) \geq 0} \qquad \forall \theta \in \Theta$$

$$\int\limits_\theta^{\overline{\theta}} q(y)dy \leq \int\limits_\theta^{\overline{\theta}} \underline{D}(y)dy \qquad \forall \theta \in \Theta \qquad \textit{and} \qquad \underline{W}(\underline{\theta}, q) \geq G^*$$

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5. **Bayesian design with conservative payoff guarantees**
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Bayesian Design under Conservative Payoff Guarantees

Bayesian Design with Conservative Payoff Guarantees

- In many settings designer **does have** model $(V^\star, F^\star)$
- However, designer needs approval from

  - regulatory authorities

  - upstream supervisor

  - ...

Bayesian Design with Conservative Payoff Guarantees

- In many settings designer **does have** model $(V^{\star}, F^{\star})$
- However, designer needs approval from

  \- regulatory authorities

  \- upstream supervisor

  \- ...
- **Approver**
  - same objective as designer
  - but **does not trust designer's confidence in model**
    - limited expertise
    - legal requirements/impediments
    - value-at-risk obligations
    - ...
  - asks for satisfactory **welfare guarantees**

Alternative Interpretation (of robustness constraint)

- Designer does not fully trust her own model
  - first prepares for worst (in case model is mis-specified)
  - then uses model to select among worst-case-optimal mechanisms
- Two interpretations equivalent

Bayesian Design with Conservative Payoff Guarantees

- Supervisor approves only mechanisms with highest welfare guarantee

Definition

Short-list (of approved mechanisms)

$$\mathcal{M}^{\mathrm{SL}} \equiv \arg\max_{M \in \mathcal{M}} G(M)$$

Bayesian Design with Conservative Payoff Guarantees

- Supervisor approves only mechanisms with highest welfare guarantee

Definition

Short-list (of approved mechanisms)

$$\mathcal{M}^{\mathrm{SL}} \equiv \arg\max_{M \in \mathcal{M}} G(M)$$

- Mechanisms with largest guarantee
- More permissive lists: $\mathcal{M}_{\gamma}^{\mathrm{SL}} \equiv \{M \in \mathcal{M} : G(M) \geq \gamma G^{*}\})$, $\gamma \in [0, 1]$

Designer's Problem

- Choose mechanism that maximizes expected welfare under conjectured model $(V^\star, F^\star)$ over SL of worst-case-optimal mechanisms

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- Choose mechanism that maximizes expected welfare under conjectured model $(V^\star, F^\star)$ over SL of worst-case-optimal mechanisms

Definition

Mechanism M **robustly optimal** if

$$M \in \arg \max_{M' \in \mathcal{M}^{\mathrm{SL}}} W(M'; V^*, F^*)$$

Robust Optimality

- **Robustly optimal schedule q^{OPT}** solves

$$\max_q \int_{\underline{\theta}}^{\overline{\theta}} \Big[V^\star(q(\theta)) - z^\star(\theta) q(\theta) \Big] F^\star(\mathrm{d}\theta)$$

$$\text{s.t.}$$

$$q \quad \text{non-increasing}$$

$$\underline{W}(\theta, q) := \underline{V}(q(\theta)) - \theta q(\theta) - \int_{\theta}^{\overline{\theta}} q(y) dy \geq G^* \qquad \forall\ \theta \in \Theta$$

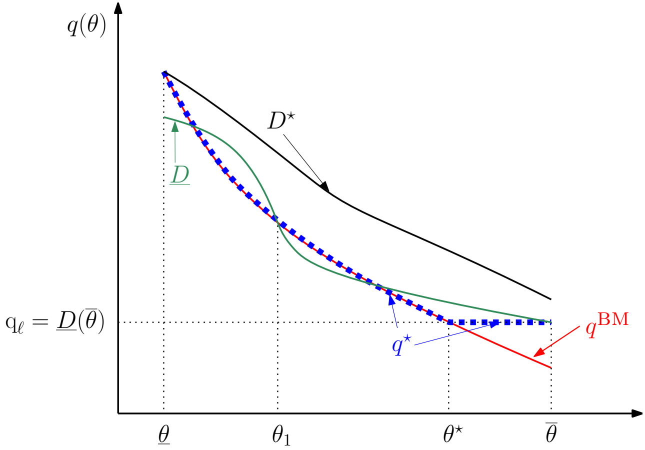
where $z^\star(\theta) := \theta + \frac{F^\star(\theta)}{f^\star(\theta)}$ is **virtual cost** under conjectured model $(V^\star, F^\star)$.

Baron-Myerson-with-Quantity-Floor

Definition

BM-with-quantity-floor:

$$q^{\star}(\theta) := \max\{q^{\mathrm{BM}}(\theta), \mathrm{q}_{\ell}\}$$



Robust Optimality of BM-with-Quantity-Floor

Proposition

BM-with-quantity-floor $M^\star = (q^\star, u^\star)$ *robustly optimal iff* $\underline{W}(\underline{\theta}, q^\star) \geq G^*$ *and*

$$\int_{\theta}^{\overline{\theta}} q^\star(y)dy \leq \int_{\theta}^{\overline{\theta}} \underline{D}(y)dy \qquad \forall\ \theta \in \Theta$$

Furthermore, when $M^\star = (q^\star, u^\star)$ *robustly optimal, any* $M^{\mathrm{OPT}} = (q^{\mathrm{OPT}}, u^{\mathrm{OPT}})$ *s.t.*

$$q^{\mathrm{OPT}}(\theta) = q^\star(\theta) \qquad \forall\ \theta > \underline{\theta}$$

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Corollary

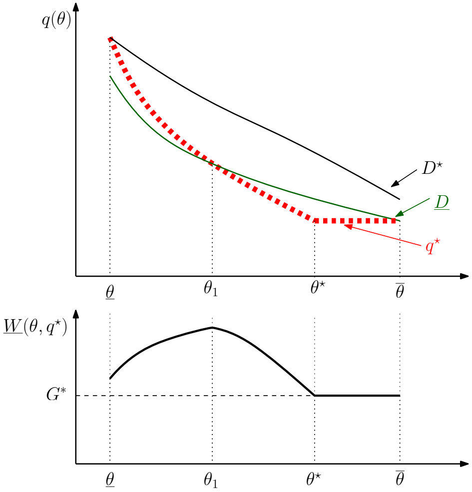
When uncertainty only over type distribution (more generally, $D^\star = \underline{D}$*)*

- *BM-with-quantity-floor robustly optimal*
- *efficiency at both bottom and top*

(BM-floor: Proof)

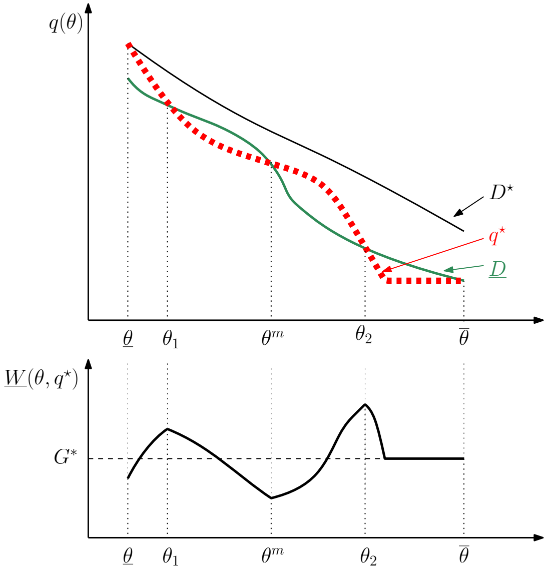
Robust Optimality of BM-with-Quantity-Floor

Suppose $D^\star \neq \underline{D}$, q^{BM} single-crosses $\underline{D}$ from above, and $\underline{W}(\underline{\theta}, q^\star) \geq G^*$. Then BM-with-quantity-floor robustly optimal

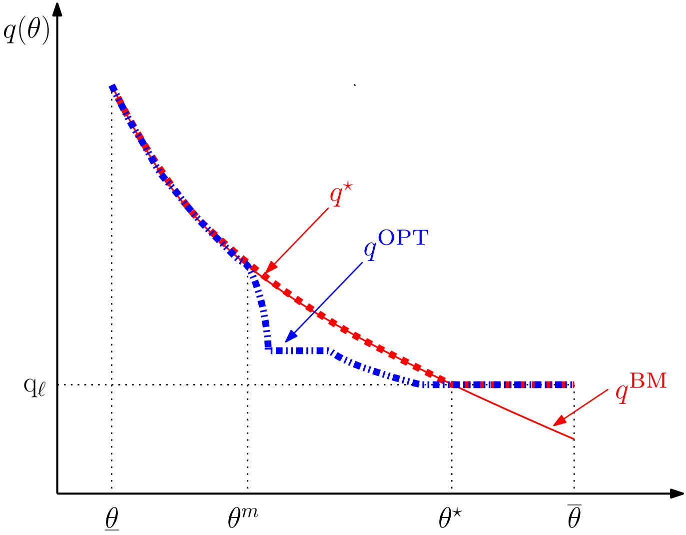


(Monotonicity Lemma)

Non-robustness of BM-with-quantity-floor



General Case



Robust Optimality: General Case

- **Reduced procurement for intermediate costs**
  - avoids over-procurement and excessive rents when $D < D^*$
- **Increased procurement for high costs**
  - avoids inefficiencies motivated by rent extraction (for lower types)
- **Low costs**
  - welfare higher than G^* even when $D < D^*$
  - no adjustments necessary

(Proposition: General Case)

(More General Cost Uncertainty)

Downstream Markets and Price Regulation

- Suppose supplier sells to **downstream mkt**
- Idea: use **price regulation** for **demand discovery**
- (Direct) price mechanism $\widetilde{M} = (p, t)$
  - **price schedule** $p : \Theta \to \mathbb{R}_+$
  - transfer schedule $t : \Theta \times \mathcal{D} \to \mathbb{R}$
  - conditioning t on realized demand D
    - irrelevant under qty regulation
    - protects seller from uncertainty under price regulation

(EPIC and EPIR)

Price Regulation: Welfare Guarantee

Lemma

For any price mechanism $\widetilde{M} \in \widetilde{\mathcal{M}}$,

$$G(\widetilde{M}) \leq G^*.$$

There exists price mechanism $\underline{\widetilde{M}} \in \widetilde{\mathcal{M}}$ *s.t.*

$$G(\underline{\widetilde{M}}) = G^*.$$

Any $\widetilde{M} \in \widetilde{\mathcal{M}}^{\mathrm{SL}}$ *(i.e., for which* $G(\widetilde{M}) = G^*$*) s.t.*

$$p(\overline{\theta}) = \overline{\theta}$$

and $\tilde{u}(\overline{\theta}, \underline{D}) = 0$.

- Maximal guarantee same as with quantity mechanisms
- Price cap at $\overline{\theta}$

Robust Optimality: Baron-Myerson-with-price-cap

- Designer's price regulation problem

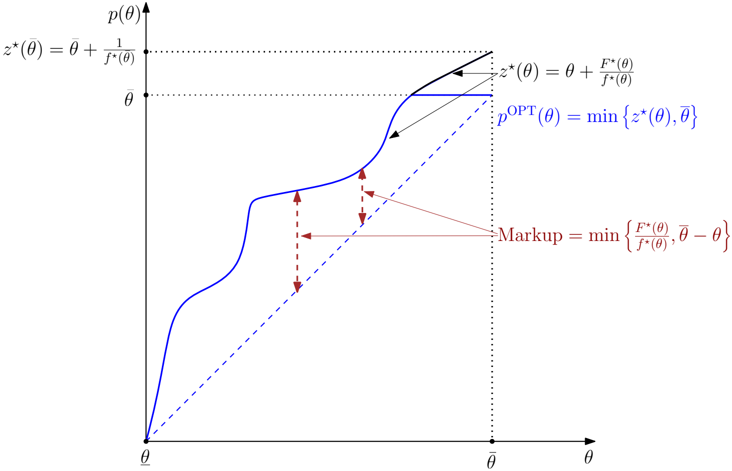
$$\max_{\tilde{M} \in \widetilde{M}^{\mathsf{SL}}} W(\tilde{M}; D^*, F^*)$$

Proposition

In any robustly optimal price mechanism $\widetilde{M}^{\mathsf{OPT}} = (p^{\mathsf{OPT}}, \tilde{u}^{\mathsf{OPT}})$

$$p^{OPT}(\theta) = \min\{z^\star(\theta), \overline{\theta}\} \qquad \forall \theta \in \Theta$$

Robust Optimality: Baron-Myerson-with-price-cap



(Proof Sketch)

Quantity vs Price Regulation?

- When is qty regulation superior to price regulation?
  - Weitzman 78: depends
  - Bayesian design: price wins
  - Worst-case optimality: tie
  - Robust optimality?

- M^{OPT} : robustly optimal **quantity** mechanism

- $\widetilde{M}^{\text{OPT}}$: robustly optimal **price** mechanism

Definition

Price regulation dominates if

$$\widetilde{W}(\widetilde{M}^{\text{OPT}}; D^\star, F^\star) \geq W(M^{\text{OPT}}; V^\star, F^\star)$$

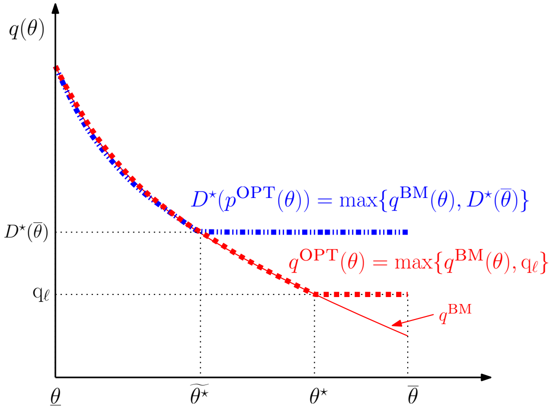
Quantity regulation dominates if

$$\widetilde{W}(\widetilde{M}^{\text{OPT}}; D^\star, F^\star) \leq W(M^{\text{OPT}}; V^\star, F^\star)$$

Quantity Regulation Dominates

Proposition

If BM-with-quantity-floor is robustly optimal, then quantity regulation dominates (strictly if $D^\star(\overline{\theta}) > \underline{D}(\overline{\theta})$).

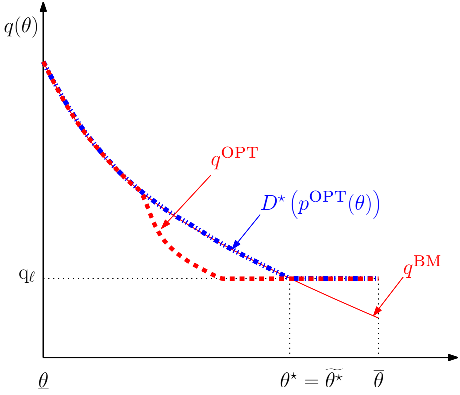


(Reasoning)

Price Regulation Dominates

Proposition

If BM-with-quantity-floor **not** *robustly optimal and $D^\star(\overline{\theta}) = \underline{D}(\overline{\theta})$, price regulation strictly dominates.*



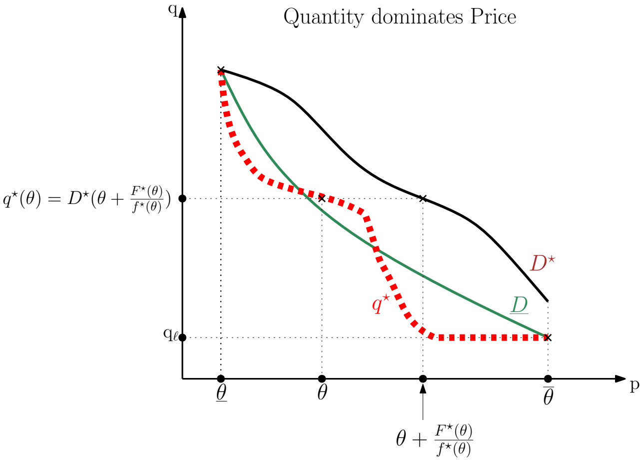
(Reasoning)

Quantity vs Price Regulation: Primitives

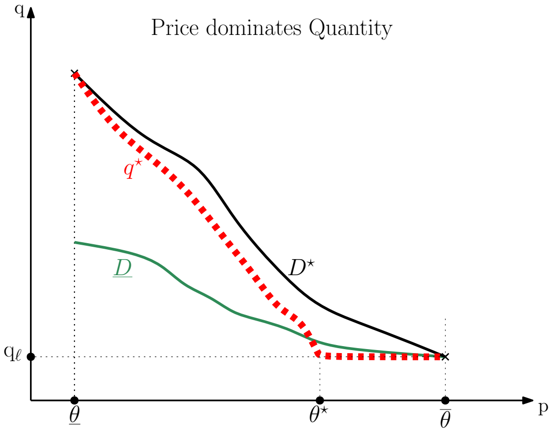
- **Qty regulation dominates** when
  - markups $F^\star(p)/f^\star(p)$ large relative to $D^* - \underline{D}$
- **Price regulation dominates** when
  - markups $F^\star(p)/f^\star(p)$ small relative to $D^* - \underline{D}$

(Proposition: Qty vs Price Regulation)

Quantity Regulation Dominates



Price Regulation Dominates



Plan

Lexicographic Robustness

Lexicographic Robustness: Kambhampati (2026)

- Max-min optimality: many (continuum of) mechanisms
- (Endogenous) lexicographic beliefs refine beyond worst case
- Properly selected belief systems **eliminate rent-extraction vs efficiency tradeoff**

Lexicographic Probability Systems

- Suppose V known to designer
- Expected payoff of M under F: $w(M;F) := W(M;V,F)$
- Uncertainty: $\mathcal{F} \subset \text{CDF}(\Theta)$
  - each $F \in \mathcal{F}$: finite support $\Theta^{\#} := (\theta_1, \theta_2, ..., \theta_N)$, with $f(\theta) := \Pr(\theta)$
- **Lexicographic probability system** (LPS): ordered list of beliefs

$$\mathrm{F} = (F_1, ..., F_K)$$

- **Lexicographic order**: Let $a, b \in \mathbb{R}^K$. Say $a \geqslant_L b$ if

$$b_k > a_k \text{ for some } k \in \{1, \cdots, K\} \implies a_j > b_j \text{ for some } j < k$$

- Type θ F-**infinitely more likely** than θ' ($\theta >_{\mathrm{F}} \theta'$) if

$$\min\{k : f_k(\theta) > 0\} < \min\{k : f_k(\theta') > 0\}$$

- $\theta \geqslant_{\mathrm{F}} \theta'$ if not true that $\theta' >_{\mathrm{F}} \theta$
- Mechanisms compared by expected payoff vector

$$w_{\mathrm{F}}(M) \equiv (w(M;F_1), ..., w(M;F_K))$$

- $M \in \mathcal{M}$ F-**optimal** iff $w_{\mathrm{F}}(M) \geq_L w_{\mathrm{F}}(M')$ all $M' \in \mathcal{M}$

Definitions

- **Definition**: F **adversarial** wrt $M = (q, t)$ if

$$f_1(\theta) > 0 \Rightarrow V(q(\theta)) - t(\theta) \leq V(q(\theta')) - t(\theta'),\ \forall\, \theta' \in \Theta^{\#}$$

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- **Definition**: F has **full support** if $\forall\, \theta \in \Theta^{\#}$, $\exists\, k \in \{1, \cdots, K\}$ s.t. $f_k(\theta) > 0$.

Definitions

- **Definition**: M **robust** if $\exists\, \mathrm{F}$ adversarial wrt M s.t. M is F-optimal.

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Definitions

- **Definition**: M **robust** if $\exists\,\mathrm{F}$ adversarial wrt M s.t. M is F-optimal.
- **Definition**: M is **perfectly robust** if $\exists\,\mathrm{F}$ adversarial wrt M with full support s.t. M is F-optimal.
- **Definition**: M is **properly robust** if $\exists\,\mathrm{F}$ strongly adversarial wrt M with full support s.t. M is F-optimal.

Definitions

- **Definition**: $M = (q^{FB}, t^{FB})$ is **FB-efficient** (with minimal transfers) if,

1. $q^{FB}(\theta) = \arg\max_{\mathrm{q} \in [0,\bar{q}]} \{V(\mathrm{q}) - \theta \mathrm{q}\}$ all $\theta \in \Theta^{\#}$

2. No rent at the bottom:

$$t^{FB}(\theta_N) = \theta_N q^{FB}(\theta_N)$$

3. Upward-adjacent IC constraints bind, i.e., all $j = 1, ..., N-1$

$$t^{FB}(\theta_j) - \theta_j q^{FB}(\theta_j) = t^{FB}(\theta_{j+1}) - \theta_j q^{FB}(\theta_{j+1})$$

Robustness and WCO

Theorem

M robust iff

1. *FB-efficiency at "bottom," i.e., with highest-cost type:* $q(\theta_N) = q^{FB}(\theta_N)$
2. $t(\theta_N) = t^{FB}(\theta_N)$
3. *lowest welfare attained at "bottom":*

$$V(q(\theta)) - t(\theta) \geq V(q(\theta_N)) - t(\theta_N) \quad \forall \theta \in \Theta^{\#}$$

Robustness and WCO

Theorem

M robust iff

1. *FB-efficiency at "bottom," i.e., with highest-cost type:* $q(\theta_N) = q^{FB}(\theta_N)$
2. $t(\theta_N) = t^{FB}(\theta_N)$
3. *lowest welfare attained at "bottom":*

$$V(q(\theta)) - t(\theta) \geq V(q(\theta_N)) - t(\theta_N) \quad \forall \theta \in \Theta^{\#}$$

- Hence, robustness implies WCO (saddle point)

(Robustness and WCO: Proof)

Perfect Robustness and Undomination

Definition: $M = (q, t)$ **undominated** if there is no IC-IR $M' = (q', t')$ s.t.

$$V(q'(\theta)) - t'(\theta) \geqslant V(q(\theta)) - t(\theta) \quad \forall \theta \in \Theta^{\#}$$

with inequality strict for some $\theta \in \Theta^{\#}$.

Theorem

*M **perfectly robust** iff (a) robust and (b) undominated.*

(Perfect Robustness and Undomination: Proof)

Proper Robustness and Efficiency

Theorem

M **properly robust** *iff FB-efficient.*

- Hence, in any properly robust mechanism:
  - **efficient procurement from all types**
  - **minimal transfer compatible with IC-IR of all types**

(Proper Robustness and Efficiency: Proof)

Plan

1. Introduction
2. Procurement
3. Bayesian design
4. Worst-case optimality
5. Bayesian design with conservative payoff guarantees
6. Lexicographic max-min
7. **Competitive ratio**
8. Min-max regret
9. Conclusions

Competitive Ratio

Competitive Ratio: Bergemann-Heumann-Morris (2025)

- Buyer knows V (equivalently, D) but has no prior over seller's cost.
- Objective: guarantee maximal fraction of efficient social surplus.
- Main result: **constant utility-share mechanism optimal**.

Environment

- Same as above, but with constant-elasticity gross value fn
$$V^*(\mathrm{q}) = \frac{\mathrm{q}^\sigma}{\sigma}, \qquad \sigma \in (0,1)$$
- Seller privately knows increasing, convex cost fn C (e.g., $C(\mathrm{q}) = \theta \mathrm{q}$)
- Designer commits to DRM (t, q) – equivalently, tariff $T : [0, \bar{\mathrm{q}}] \to \mathbb{R}$
- Given (C, T), seller chooses $\mathrm{q} \in [0, \bar{\mathrm{q}}]$ to max $T(\mathrm{q}) - C(\mathrm{q})$
- Let
$$Q(C, T) \in \arg\max_{\mathrm{q} \in [0, \bar{\mathrm{q}}]} \{T(\mathrm{q}) - C(\mathrm{q})\}.$$
- Designer's ex-post payoff: $U(C, T) := V(Q(C, T)) - T(Q(C, T))$.

Competitive Ratio

- Complete-information efficient surplus:

$$W(C) := \max_{q \in [0, \bar{q}]} \{V(q) - C(q)\}$$

- Relative performance of T at cost C:

$$R(C, T) := \frac{U(C, T)}{W(C)}.$$

Definition

Competitive ratio is

$$\sup_{T \in \mathcal{T}} \inf_{C \in \mathcal{C}} R(C, T).$$

Constant-Share Mechanism

Definition

Constant-share mechanism pays

$$T_z(\mathrm{q}) = z\,V(\mathrm{q}), \qquad z \in (0,1).$$

- Buyer keeps $(1-z)V(\mathrm{q})$.

Linear Costs: Exact Competitive Ratio

- Suppose admissible set $\mathcal{C}$ s.t., for any $C \in \mathcal{C}$, there exists $\theta \in \Theta$ s.t. for all $\mathrm{q} \in [0, \bar{\mathrm{q}}]$,

$$C(\mathrm{q}) = \theta \mathrm{q}$$

- Then,

$$Q(C, T_z) = \left(\frac{z}{\theta}\right)^{1/(1-\sigma)}$$

and

$$R(C, T_z) = \frac{U(C, T_z)}{W(C)} = \frac{1-z}{1-\sigma}\frac{1}{z^{\sigma/(1-\sigma)}},$$

which is independent of θ.

Theorem 1

Optimal constant-share mechanism is $z^* = \sigma$, and

$$\sup_T \inf_{C \in \mathcal{C}} \frac{U(C, T)}{W(C)} = \sigma^{\frac{\sigma}{1-\sigma}}.$$

Linear Costs: Exact Competitive Ratio

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Theorem 1

Optimal constant-share mechanism is $z^* = \sigma$, and
$$\sup_{T} \inf_{C \in \mathcal{C}} \frac{U(C, T)}{W(C)} = \sigma^{\frac{\sigma}{1-\sigma}}.$$

- No mechanism, linear or not, improves over optimal constant-share one
- Guarantee falls from 1 to $1/e$ as demand becomes more elastic
- Optimal constant-share mech procures $\sigma^{1/(1-\sigma)}$ of efficient output $Q^{FB}(C) = \theta^{\frac{1}{\sigma-1}}$ in each state: **distortions for all types** θ

Why Constant-Share Optimal?

- Non-constant transfer rule favors some output scales over others.
- Nature selects linear cost whose optimal output $Q(C, T)$ yields low scale.
- Constant sharing equalizes relative performance across scales.
- Constant-share mechanism Bayes-optimal for cdf $F(\theta) = (\theta/\bar{\theta})^{\alpha}$ on $[0,\bar{\theta}]$, with $\alpha \to \sigma/(1-\sigma)$

General Convex Costs

- With arbitrary increasing convex costs C, constant-share mechanism T_z with $z = \sigma$ no longer exactly optimal.
- Let $z^*(\sigma)$ maximize worst-case ratio, i.e.,

$$z^*(\sigma) \in \arg\max_{z\in[0,1]} \inf_{C\in\mathcal{C}} \frac{U(C, T_z)}{W(C)}$$

- Let

$$B(\sigma) := \frac{1 - z^*(\sigma)}{(1-\sigma)(z^*(\sigma))^{\sigma/(\sigma-1)} + \sigma z^*(\sigma)} = \inf_{C\in\mathcal{C}} \frac{U(C, T_{z^*(\sigma)})}{W(C)}.$$

Theorem

For every mechanism T,

$$\inf_{C\in\mathcal{C}} \frac{U(C, T)}{W(C)} \leq B(\sigma),$$

with equality iff $T = T_{z^*(\sigma)}$.

Elasticity Governs Guarantee

- Optimal share $z^*(\sigma)$ increases with σ: elastic demand requires stronger supply incentives.
- Numerically, $z^*(\sigma)$ stays close to σ:

$$|z^*(\sigma) - \sigma| \leq 0.12.$$

- Guarantee declines with σ and is close to linear:

$$|B(\sigma) - (1 - \sigma)| \leq 0.16.$$

- Unlike linear-cost case, $\lim_{\sigma \to 1} B(\sigma) = 0$: flexible value functions permit adversarial Nature to grasp most surplus

Plan

Min-max Regret

Min-Max Regret: Guo-Shmaya (2025)

- Monopolist sells quantity $\mathrm{q} \in [0, \bar{q}]$, with $\bar{q} = 1$, in downstream market w. inverse demand P s.t. $P(0) = \bar{v}$.
- Monopolist observes
  - inverse demand P (decreasing and upper semicontinuous)
  - cost fn C (increasing, lower semicontinuous, with $C(0) = 0$)
- Designer (regulator) faces uncertainty over (P, C)
- Designer's policy $\rho(\mathrm{q}, \mathrm{p})$ specifies monopolist's net revenue when transacting at (q, p)
- Taxation Principle: any outcome of any IC-IR DRM $M = (q(P, C), t(P, C))$ implementable with tariff ρ
- Revenue ρ funded with consumer's money
- Given (C, P) and ρ, monopolist chooses (q, p), with $\mathrm{p} \leq P(\mathrm{q})$ to maximize

$$\rho(\mathrm{q}, \mathrm{p}) - C(\mathrm{q}).$$

Objective and Regret

- Regulator's payoff
$$RS_\alpha = CS + \alpha\,\Pi, \qquad \alpha \in [0,1].$$
- $\alpha = 0$: consumers' surplus
- $\alpha = 1$: total surplus

Objective and Regret

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- $\alpha = 1$: total surplus

Definition 2

Given (P, C), ex-post regret under ρ

$$\text{Regret} = CIP(P,C) - RS_\alpha(\rho, P, C; q^*(\rho, P, C), p^*(\rho, P, C)),$$

where $CIP(P,C)$ is designer's complete-information payoff and (q^*, p^*) is firm's best response to ρ given (P, C).

Objective and Regret

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Optimal policy ρ^* minimizes ex-post regret over all (P, C) in admissible set $\mathcal{A} = \mathcal{P} \times \mathcal{C}$ and all firm's best responses (q^*, p^*)

Three Sources of Regret

1. **Rents to firm:** high revenue transfers surplus from consumers to firm.
2. **Underproduction:** weak marginal incentives leave valuable consumers unserved.
3. **Overproduction:** subsidies may induce output whose cost exceeds consumer value.

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1. **Rents to firm:** high revenue transfers surplus from consumers to firm.
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Designer problem

No single instrument controls all three margins. Optimal policy combines

- average-revenue cap
- piece-rate subsidy
- cap on total subsidy.

Worst-Case Regret

- Let $f_\alpha \equiv 1/(2-\alpha)$ and

$$r_\alpha = \max_{\mathrm{q}\in[0,1],\mathrm{p}\in[0,f_\alpha \bar{v}],} \min\left\{\mathrm{q}(1-f_\alpha)\bar{v} - \mathrm{qp}\log \mathrm{q}; \mathrm{q}(f_\alpha \bar{v} - \mathrm{p})\right\}.$$

Theorem

Every policy has worst-case regret greater or equal to r_α, and bound is attainable.

Worst-Case Regret

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Theorem

Every policy has worst-case regret greater or equal to r_α, and bound is attainable.

As α rises, redistribution matters less and regulator tolerates higher firm revenue to improve output incentives.

Polar Case: Consumer Surplus

- When $\alpha = 0$ (i.e., designer maximizes CS), $f_0 = 1/2$ and optimal policy is
$$\rho(\mathrm{q}, \mathrm{p}) = \min \left\{ \frac{\mathrm{q}\bar{v}}{2}, \ \mathrm{qp} \right\}.$$
- Equivalently, **price cap** at $\bar{v}/2$.
- Cap limits firm's extraction of consumers' surplus.
- No subsidy needed because firm's profit receives zero welfare weight.

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- Cap limits firm's extraction of consumers' surplus.
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Implication: **min-max-regret foundation for widely used regulatory instrument**.

Polar Case: Total Surplus

- When $\alpha = 1$ (i.e., designer maximizes TS), $f_1 = 1$ and optimal policy is

$$\rho(\mathrm{q}, \mathrm{p}) = \min\{\mathrm{q}\bar{v},\ \mathrm{qp} + r_1\}.$$

- Equivalently, **piece-rate subsidy** $(\bar{v} - \mathrm{p})$ with **total subsidy cap** at r_1.

Polar Case: Total Surplus

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- Equivalently, **piece-rate subsidy** $(\bar{v} - \mathrm{p})$ with **total subsidy cap** at r_1.

Trade-off

Subsidy mitigates underproduction at low quantities; subsidy cap prevents large losses from overproduction.

Optimal Policy for Any Welfare Weight

- Let

$$s_\alpha := \left(\sup \mathrm{q}(f_\alpha \bar{v} - \mathrm{p}) : q(1 - f_\alpha)\bar{v} - \mathrm{qp} \log \mathrm{q} > r_\alpha,\ (\mathrm{q}, \mathrm{p}) \in [0,1] \times [0, f_\alpha \bar{v}]\right)^+$$

be smallest subsidy cap consistent with regret r_α.

Theorem

For every $s \in [s_\alpha, r_\alpha]$,

$$\rho(\mathrm{q}, \mathrm{p}) = \min\{\mathrm{q} f_\alpha \bar{v},\ \mathrm{qp} + s\}$$

achieves worst-case regret r_α.

Optimal Policy for Any Welfare Weight

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Theorem

For every $s \in [s_\alpha, r_\alpha]$,

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achieves worst-case regret r_α.

Optimal policy uses same instruments as in polar cases $\alpha = 0, 1$, i.e.,

- **piece-rate subsidy** $(f_\alpha \bar{v} - \mathrm{p})$ at low prices (prevents underproduction)
- **cap on average revenue** at $f_\alpha \bar{v}$ (prevents excessive CS extraction)
- **cap on total subsidy** at s (prevents overproduction)

Additional Knowledge

- Suppose all $P \in \mathcal{P}$ s.t. $\underline{P} \leq P \leq \overline{P}$.
- Let

$$\underline{V}(\mathrm{q}) = \int_0^{\mathrm{q}} \underline{P}(z)dz \qquad \text{and} \qquad \overline{V}(\mathrm{q}) = \int_0^{\mathrm{q}} \overline{P}(z)dz$$

  and note that (p, q) reveals that CS is at least

$$\Theta(\mathrm{q}, \mathrm{p}) = \int_0^{\mathrm{q}} \max\{\underline{P}(z), \mathrm{p}\}dz.$$

Theorem

Optimal policy is

$$\rho(\mathrm{q}, \mathrm{p}) = \min\{f_\alpha \overline{V}(\mathrm{q}),\ \Theta(\mathrm{q}, \mathrm{p}) + s - d(\mathrm{q})\}$$

where

$$d(\mathrm{q}) := \max_{0 \leq z \leq \mathrm{q}} \{\underline{V}(z) - f_\alpha \bar{V}(z)\}$$

(with s appropriately chosen).

- Policy mitigates rents while avoiding too much (too little) production

Additional Knowledge

- With no uncertainty over demand (i.e., $\underline{P} = \overline{P} = P$)

$$\rho(\mathrm{q}, \mathrm{p}) = f_\alpha \overline{V}(\mathrm{q})$$

  i.e., revenue is constant fraction f_α of gross consumer value (as with competitive ratio).

Conclusions

Conclusions

- Robust contracting
  - central to many design problems
- Traditional approach to robustness: worst-case optimality
  - multiple (continuum of) WCO mechanisms, many dominated
- Other model-free approaches deliver sharper predictions
  - competitive ratio
  - min-max regret
  - lexicographic min-max
- **Bayesian design subject to guarantee constraints**
  - blends ideas/techniques from Bayesian design with robustness analysis
  - designer uses model/conjecture to select over mechanisms yielding satisfactory guarantee

Conclusions

- A lot remains to be done
  - dynamics and experimentation (Suraj's lecture)
  - multi-dimensional screening (Rahul's lecture)
  - many agents and information robustness (Ben's lecture)
  - robust control, micro-foundations, and macro-finance (Lars' lecture)
  - value at risk and macro (Cosmin's lecture)
  - competing designers (largely unexplored)
  - ...

Most Important Slide

THANK YOU

Baron Myerson: Proof

Lemma

Mechanism $M = (q, u)$ IC and IR iff

1. q non-increasing
2. for all $\theta \in \Theta$,

$$u(\theta) = u(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} q(z)dz$$

3. $u(\overline{\theta}) \geq 0$

Baron Myerson: Proof

- Expected payoff of any IC and IR $M \in \mathcal{M}$ under model $(V^\star, F^\star)$:

$$\int_{\underline{\theta}}^{\overline{\theta}} \Big[V^\star(q(\theta)) - z^\star(\theta) q(\theta) \Big] F^\star(\mathrm{d}\theta) - u(\overline{\theta})$$

- $q^{BM}(\theta)$ maximizes virtual surplus $V^\star(\mathrm{q}) - z^\star(\theta)\mathrm{q}$, all θ

- Regularity:

  $\Rightarrow q^{BM}$ decreasing

  $\Rightarrow q^{BM}$ (along with $u^{BM}(\overline{\theta}) = 0$ and $u(\theta) = \int_{\theta}^{\overline{\theta}} q(z)dz$) **SEU-optimal**

Back

Max-guarantee-proof

For any IC and IR mechanism $M = (q, u)$

$$
\begin{aligned}
W(M; V, F) &:= \int \{V(q(\theta)) - \theta q(\theta) - u(\theta)\} F(\mathrm{d}\theta) \\
&\geq \int \{\underline{V}(q(\theta)) - \theta q(\theta) - u(\theta)\} F(\mathrm{d}\theta) \\
&\geq \inf_{\theta\in\Theta} \left[\underline{V}(q(\theta)) - \theta q(\theta) - u(\theta)\right]
\end{aligned}
$$

Hence,

$$G(M) \geq \inf_{\theta\in\Theta} \left[\underline{V}(q(\theta)) - \theta q(\theta) - u(\theta)\right]$$

Because $\underline{V} \in \mathcal{V}$ and, for each θ, Dirac distribution at θ is in $\mathcal{F}$

$$G(M) \leq \inf_{\theta\in\Theta} \left[\underline{V}(q(\theta)) - \theta q(\theta) - u(\theta)\right]$$

Hence,

$$G(M) = \inf_{\theta\in\Theta} \left[\underline{V}(q(\theta)) - \theta q(\theta) - u(\theta)\right]$$

Because $u(\overline{\theta}) \geq 0$ and

$$\underline{V}(q(\overline{\theta})) - \overline{\theta} q(\overline{\theta}) \leq \underline{V}(\mathrm{q}_\ell) - \overline{\theta}\mathrm{q}_\ell := G^*$$

$G(M) \leq G^*$

Back

BM-with-floor: Proof

- Full program:

$$\max_{q} \int_{\underline{\theta}}^{\overline{\theta}} \left[V^\star(q(\theta)) - z^\star(\theta) q(\theta) \right] F^\star(\mathrm{d}\theta)$$

$$q \quad \text{non-increasing}$$

$$\underline{W}(\theta; q) := \underline{V}(q(\theta)) - \theta q(\theta) - \int_{\theta}^{\overline{\theta}} q(y) dy \geq G^* \qquad \forall \theta \in \Theta$$

BM-with-floor: Proof

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$$\underline{W}(\theta; q) := \underline{V}(q(\theta)) - \theta q(\theta) - \int_{\theta}^{\overline{\theta}} q(y) dy \geq G^{*} \qquad \forall \theta \in \Theta$$

- **Relaxed program:**

$$\max_{q} \int_{\underline{\theta}}^{\overline{\theta}} \Big[V^{\star}(q(\theta)) - z^{\star}(\theta) q(\theta) \Big] F^{\star}(\mathrm{d}\theta)$$

$$q(\theta) \geq \mathrm{q}_{\ell} \qquad \forall \theta \in \Theta$$

BM-with-floor: Proof

- Full program:

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$$q(\theta) \geq \mathrm{q}_\ell \qquad \forall \theta \in \Theta$$

- BM-with-quantity-floor schedule $q^\star$ solves relaxed program
- Majorization lemma: conditions in Prop $\Rightarrow$ $\underline{W}(\theta; q^\star) \geq G^*$ all θ

Back

Monotonicity Lemma

- Recall

$$\underline{W}(\theta, q) := \underline{V}(q(\theta)) - \theta q(\theta) - \int_{\theta}^{\overline{\theta}} q(y)dy$$

Lemma (Monotonicity)

For any non-increasing q and interval $I \subseteq \Theta$,

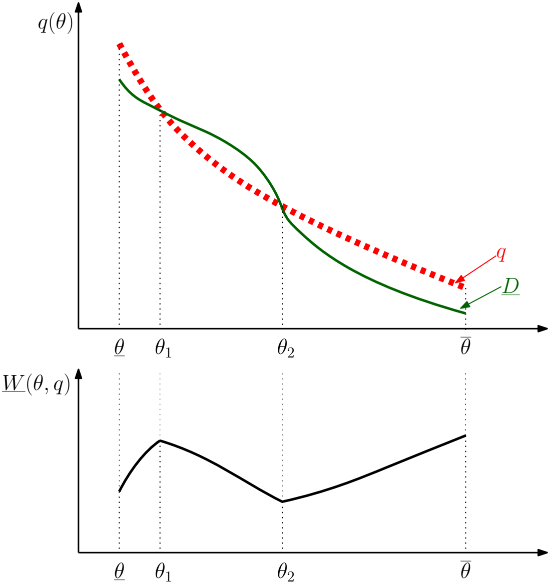
1. *If $0 < q(\theta) \leq \underline{D}(\theta)$ for all $\theta \in I$, then $\underline{W}(\cdot, q)$ non-increasing over I*

   (decreasing if q decreasing and $q(\theta) < \underline{D}(\theta)$ all $\theta \in I$)

2. *If $q(\theta) > \underline{D}(\theta)$ for all $\theta \in I$, then $\underline{W}(\cdot, q)$ non-decreasing over I*

   (increasing if q decreasing)

Monotonicity Lemma: Illustration



Proposition: General Case

Let

$$\theta^m := \max\{\arg\min_{\theta\in\Theta} \underline{W}(\theta, q^\star)\}$$

$$\theta^\star := \begin{cases} (q^{\mathrm{BM})^{-1}}(\mathrm{q}_\ell) & \text{if} \quad q^{\mathrm{BM}}(\overline{\theta})<\mathrm{q}_\ell \\ \overline{\theta} & \text{otherwise} \end{cases}$$

Proposition

When BM-with-quantity-floor **not** *robustly optimal,*

(a) $q^{\mathrm{OPT}}(\theta) = q^{\mathrm{BM}}(\theta)$ *for all* $\theta \in (\underline{\theta}, \theta^m)$

(b) $q^{\mathrm{OPT}}(\theta) \leq q^{\mathrm{BM}}(\theta)$ *for all* $\theta \in (\theta^m, \theta^\star)$

(c) $q^{\mathrm{OPT}}(\theta) = \mathrm{q}_\ell$ *for all* $\theta \in [\theta^\star, \overline{\theta}]$

Furthermore, if $\theta^\star < \overline{\theta}$*, then* $q^{\mathrm{OPT}}(\theta) < q^{\mathrm{BM}}(\theta)$ *for all* $\theta \in (\theta^m, \theta^\star)$*. If, instead,* $\theta^\star = \overline{\theta}$*, there exists* $\theta^{\star\star} \in [\theta^m, \theta^\star]$ *such that* $q^{\mathrm{OPT}}(\theta) < q^{\mathrm{BM}}(\theta)$ *for all* $\theta \in (\theta^m, \theta^{\star\star})$ *and* $q^{\mathrm{OPT}}(\theta) = \underline{D}(\theta)$ *for all* $\theta \in [\theta^{\star\star}, \overline{\theta}]$*.*

Back

More General Cost Uncertainty

- Suppose there exist $\underline{F} \in \text{CDF}(\Theta)$ s.t.

$$\mathcal{F} = \{F \in \text{CDF}(\Theta) : \underline{F}(\theta) \leq F(\theta), \quad \forall\, \theta\}$$

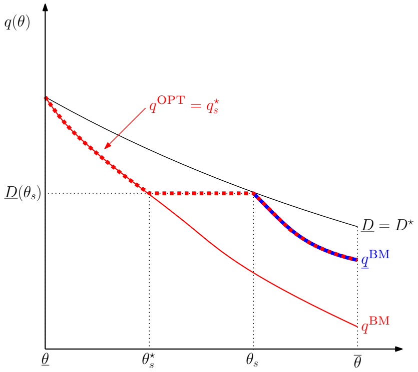
  with $F^\star \in \mathcal{F}$

- So far, $\underline{F}(\theta) = \mathbb{I}(\theta \geq \overline{\theta})$

- Generalization also accommodates for unbounded Θ

More General Cost Uncertainty

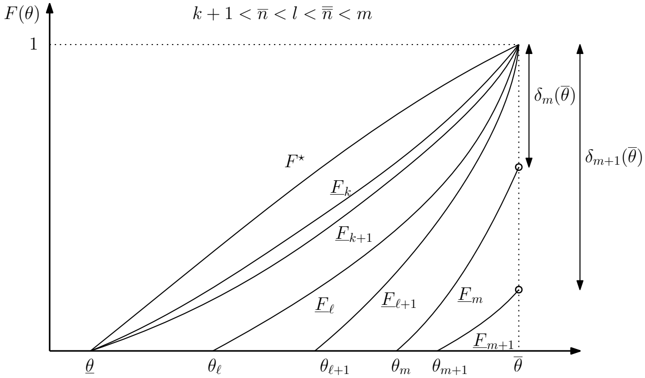
- Suppose $\underline{F}(\theta) = 0$ for all $\theta \leq \theta_s$, with $\theta_s \in (\underline{\theta}, \overline{\theta})$, and regular over $[\theta_s, \overline{\theta})$
- For simplicity, $D^\star = \underline{D}$



- Optimal mechanism bridges two BM schedules
  - Low costs: BM under $(F^*, \underline{V})$
  - High costs: BM under $(\underline{F}, \underline{V})$
  - No distortion at top **and middle** (θ_s)

Variations in Cost Uncertainty

- Suppose $\overline{\theta}$ finite
- (Monotone) sequence $(\underline{F}_n)$ converging to $\mathbb{I}(\theta \geq \overline{\theta})$



Variations in Cost Uncertainty

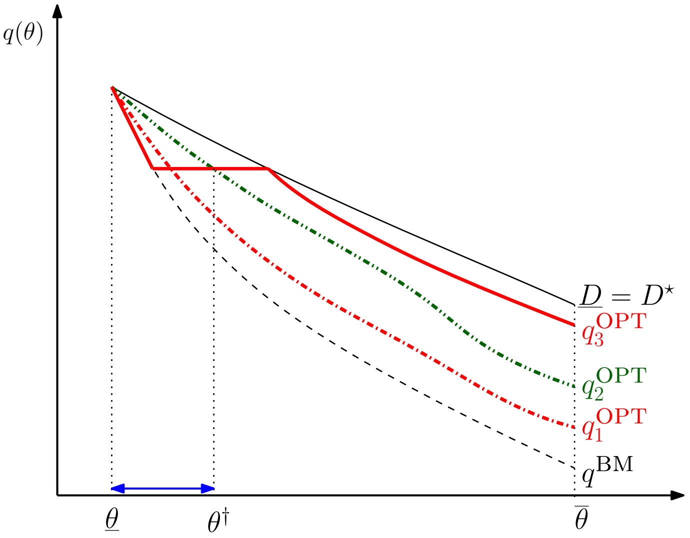
Proposition

Suppose $D^{\star} = \underline{D}$. There exist monotone sequences $(\underline{F}_n)$ s.t., for every $\theta \in (\underline{\theta}, \overline{\theta})$, there exists $n(\theta) \in \mathbb{N}$ s.t.

$$q_{n+1}^{OPT}(\theta) \left\{ \begin{array}{c} \geq \\ \leq \end{array} \right\} q_n^{OPT}(\theta) \;\; \textit{for} \;\; \left\{ \begin{array}{c} n \leq n(\theta) - 1 \\ n > n(\theta) \end{array} \right\}$$

Moreover, for all θ,there exists $j, k \in \mathbb{N}$ with $j < k$, s.t. $q_k^{OPT}(\theta) < q_j^{OPT}(\theta)$

Variations in Cost Uncertainty



Back

Price Mechanisms: EPIC and EPIR

Definition

Price mechanism $\widetilde{M} = (p, t)$ **EPIC** and **EPIR** if, for all $\theta, \theta' \in \Theta$ and $D \in \mathcal{D}$,

$$t(\theta, D) - \theta D(p(\theta)) \geq t(\theta', D) - \theta D(p(\theta'))$$

$$\tilde{u}(\theta, D) := t(\theta, D) - \theta D(p(\theta)) \geq 0$$

- $\widetilde{\mathcal{M}}$: set of EPIC and EPIR price mechanisms $\widetilde{M} := (p, \tilde{u})$

Price Mechanisms: EPIC and EPIR

Lemma

$\widetilde{M} = (p, t)$ is EPIC and EPIR iff p is non-decreasing, and for every $\theta \in \Theta$ and $D \in \mathcal{D}$,

$$\tilde{u}(\theta, D) = \tilde{u}(\overline{\theta}, D) + \int_{\theta}^{\overline{\theta}} D(p(y))dy,$$

with $\tilde{u}(\overline{\theta}, D) \geq 0$.

Back

Robust Optimality of Baron-Myerson-with-price-cap

- Welfare under $\widetilde{M} = (p, \tilde{u}) \in \widetilde{\mathcal{M}}$ given (D, F):

$$\widetilde{W}(\widetilde{M}; D, F) := \int \{V(D(p(\theta))) - \theta D(p(\theta)) - \tilde{u}(\theta, D)\} \, F(\mathrm{d}\theta)$$

Robust Optimality of Baron-Myerson-with-price-cap

$$\max_{p,\{\tilde{u}(\overline{\theta},D)\geq 0\}} \int_{\underline{\theta}}^{\overline{\theta}} \Big[V^\star(D^\star(p(\theta))) - z^\star(\theta)D^\star(p(\theta))\Big] F^\star(\mathrm{d}\theta) - \tilde{u}(\overline{\theta}, D^\star)$$

$$\text{s.t.}$$

$$p \quad \text{non-decreasing}$$

$$\int_0^{D(p(\theta))} D^{-1}(y)dy - \theta D(p(\theta)) - \int_{\theta}^{\overline{\theta}} D(p(y))dy - \tilde{u}(\overline{\theta}, D) \geq G^* \quad \forall\ \theta \in \Theta \ \ \forall\ D \in \mathcal{D}$$

$$p(\overline{\theta}) = \overline{\theta}$$

Robust Optimality of Baron-Myerson-with-price-cap

- Relaxed program:

$$\max_{p,\{\tilde{u}(\overline{\theta},D)\geq 0\}} \int_{\underline{\theta}}^{\overline{\theta}} \Big[V^\star(D^\star(p(\theta))) - z^\star(\theta)D^\star(p(\theta))\Big] F^\star(\mathrm{d}\theta) - \tilde{u}(\overline{\theta},D^\star)$$

$$\text{s.t.}$$

$$p \quad \text{non-decreasing}$$

$$\int_0^{D(\overline{\theta})} D^{-1}(y)dy - \overline{\theta}D(\overline{\theta}) - \tilde{u}(\overline{\theta},D) \geq G^* \qquad \forall\ D \in \mathcal{D}$$

$$p(\overline{\theta}) = \overline{\theta}$$

- $p^{\mathrm{OPT}}(\theta) = \min\{z^\star(\theta), \overline{\theta}\}$, along with$\{\tilde{u}^{\mathrm{OPT}}(\overline{\theta},D)\}$ chosen to satisfy remaining robustness constraints (blue inequalities), solves relaxed program

- Because $p^{\mathrm{OPT}}(\theta) \geq \theta$ all θ, remaining robustness constraint satisfied

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Quantity Regulation Dominates

- Robustness under price mechanism $\Rightarrow$ **price cap** at $\overline{\theta}$
- When $D^{\star}(\overline{\theta}) > \underline{D}(\overline{\theta})$

  $\Rightarrow$ **over-procurement (vis-a-vis quantity mechanism) for high costs**
- When BM-with-quantity-floor mechanism robustly optimal

  $\Rightarrow$ **quantity regulation dominates**

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Price Regulation Dominates

- When BM-with-quantity-floor mechanism **not** robustly optimal

  $\Rightarrow$ **downward qty adjustments** for intermediate costs (under D^* vis-a-vis q^{BM})

  - adjustments necessary to avoid over-procurement when $D < D^\star$
  - under price regulation, demand adjusts itself
  - adjustment do not occur when $D = D^*$

- If price cap at $\overline{\theta}$ does not induce over-procurement vis-a-vis q_ℓ

  $\Rightarrow$ **price regulation strictly dominates**

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Qty vs Price: Primitives

Let $\underline{\delta} \equiv \inf\{D^*(\theta) - \underline{D}(\theta) : \theta \in \Theta\}$, $\bar{\delta} \equiv \sup\{D^*(\theta) - \underline{D}(\theta) : \theta \in \Theta\}$,
$\underline{f} \equiv \inf\{f^\star(\theta) : \theta \in \Theta\}$ and $\bar{f} \equiv \sup\{f^\star(\theta) : \theta \in \Theta\}$.

Proposition

Suppose $D^\star$ is bi-Lipschitz continuous over $[0, P^\star(0)]$ with constants $0 < k \leq K < \infty$.

- *Quantity regulation dominates if, for all $\theta \in \Theta$,*

$$\frac{F^\star(\theta)}{f^\star(\theta)} \geq \frac{D^*(\theta) - \underline{D}(\theta)}{k},$$

(strictly if $D^*(\bar{\theta}) - \underline{D}(\bar{\theta}) > 0$)

- *When, instead, for all $\theta \in \Theta$,*

$$\frac{F^\star(\theta)}{f^\star(\theta)} \leq \frac{D^*(\theta) - \underline{D}(\theta)}{K\left(1 + \frac{\bar{\delta}}{\underline{\delta}}\sqrt{\frac{2\bar{f}}{\underline{f}}}\right)},$$

and $\underline{\delta} > 0$, price regulation strictly dominates.

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Robustness and WCO: Proof

- **Sufficiency** ($\Leftarrow$): Take $\mathrm{F} = (\delta_N)$, where δ_N is Dirac's delta at θ_N. Then, F is adversarial wrt M. Furthermore, M is F-optimal. Hence M robust.

Robustness and WCO: Proof

- **Necessity** ($\Rightarrow$): Suppose $V(q(\theta)) - t(\theta) < V(q^{FB}(\theta_N)) - t^{FB}(\theta_N)$ for some $\theta \in \Theta^{\#}$. Then, under any F adversarial wrt M, first-order payoff

$$w(M; F_1) < V(q^{FB}(\theta_N)) - t^{FB}(\theta_N).$$

  Consider "flat-mechanism"

$$\widetilde{M} = (\tilde{q}, \tilde{t}) \text{ s.t. } \tilde{q}(\theta) = q^{FB}(\theta_N) \text{ and } \tilde{t}(\theta) = t^{FB}(\theta_N) \quad \forall\, \theta \in \Theta^{\#}$$

  $\widetilde{M}$ is IC and IR and $w_{\mathrm{F}}(\tilde{M}) >_L w_{\mathrm{F}}(M)$. Hence M not robust.

- Furthermore, if $(q(\theta_N), t(\theta_N)) \neq (q^{FB}(\theta_N), t^{FB}(\theta_N))$, Nature can bring designer's payoff strictly below $V(q^{FB}(\theta_N)) - t^{FB}(\theta_N)$, and hence M not WCO (and therefore not robust).

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Perfect Robustness and Undomination: Proof

- **Necessity** ($\Rightarrow$): Perfect robustness means there exists full-support F adversarial wrt M s.t . M is F-optimal.

  - This means M robust.

  - Suppose M dominated (say by M'). Then $w_{\mathrm{F}}(M') >_{\mathrm{L}} w_{\mathrm{F}}(M)$ for any F of full support. Hence, M not perfectly robust.

Perfect Robustness and Undomination: Proof

- **Sufficiency** ($\Leftarrow$): Suppose M robust and undominated. Then

$$\begin{cases} V(q(\theta)) - t(\theta) \geq V(q(\theta_N)) - t(\theta_n) & \forall \theta \in \Theta^{\#} \\ (q(\theta_N), t(\theta_N)) = (q^{FB}(\theta_N), t^{FB}(\theta_N)) \end{cases}$$

- This means that Dirac δ_N at θ_N is adversarial wrt M and, for any IC-IR $M' \in \mathcal{M}$,

$$V(q'(\theta_N)) - t'(\theta_N) \leqslant V(q^{FB}(\theta_N)) - t^{FB}(\theta_N)$$

(Hence, M' delivers weakly less than M against δ_N)
- Because M undominated, $\exists$ full-support $\tilde{F} \in \Delta(\Theta^{\#})$ s.t., for any IC-IR $M' \in \mathcal{M}$,

$$\sum_{\theta \in \Theta} f(\theta) \cdot (V(q(\theta)) - t(\theta)) \geq \sum_{\theta \in \Theta} f(\theta) \cdot \big(V(q'(\theta)) - t'(\theta)\big)$$

(Proof requires some work based on linear programming).
- This means M perfectly robust (it suffices to take LPS $\mathrm{F} = (\delta_N, \tilde{F})$)

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Proper Robustness and Efficiency: Proof

- **Sufficiency**: Let $\mathrm{F} = (\delta_N, \cdots, \delta_1)$ and note that F is strongly adversarial wrt M^{FB} (because $V(q^{FB}(\theta)) - t^{FB}(\theta))$ is decreasing in θ) and has full support. It thus suffices to show that M^{FB} is F-optimal.

  Proof by induction. No IC-IR mechanism yields higher profits against δ_N. Furthermore, any mechanism $M \in \mathcal{M}$ that yields same payoff as M^{FB} against δ_N is s.t.

  - $(q(\theta_N), t(\theta_N)) = (q^{FB}(\theta_N), t^{FB}(\theta_N))$
  - leaves rent $u(\theta_{N-1})$ to θ_{N-1} no smaller than $t^{FB}(\theta_{N-1}) - \theta_{N-1} q^{FB}(\theta_{N-1})$; This because

  $$(q^{FB}(\theta_{N-1}), t^{FB}(\theta_{N-1})) = \arg\max_{\mathrm{q},\mathrm{t}} \{V(\mathrm{q}) - \theta_{N-1}\mathrm{q} - u(\theta_{N-1})\}$$
  $$\text{s.t. } u(\theta_{N-1}) \geqslant (\theta_N - \theta_{N-1}) q^{FB}(\theta_N)$$

  - Inductively, $w_{\mathrm{F}}(M^{FB}) \geqslant_L w_{\mathrm{F}}(M)$ for all IC-IR $M \in \mathcal{M}$.
  - This means M^{FB} properly robust.

Proper Robustness and Efficiency: Proof

- **Sufficiency (intuition):** Under M^{FB}, each type supplies efficiently and receives minimal transfer among all mechanisms implementing efficient trade. Hence, any IC-IR mechanism delivering higher payoff against some type $\theta_k < \theta_N$ must deliver smaller rent to θ_k. But this means lower payoff against some less-efficient type $\theta_l > \theta_k$. Because these less-efficient types are infinitely more likely under $\mathrm{F} = (\delta_N, \cdots, \delta_1)$, M^{FB} F-optimal against $\mathrm{F} = (\delta_N, \cdots, \delta_1)$.

Proper Robustness and Efficiency: Proof

Necessity follows from following 3 results:

1. $M = (q,t) \in \mathscr{M}$ properly robust $\implies$ $V(q(\theta)) - t(\theta)$ non-increasing.

2. $M = (q,t) \in \mathscr{M}$ properly robust $\implies$ M robust.

3. $M = (q,t) \in \mathscr{M}$ properly robust and $(q(\theta_i), t(\theta_i)) = (q^{FB}(\theta_i), t^{FB}(\theta_i))$ for all $i = N-k, ..., N$ $\implies$

$$(q(\theta_{N-k-1}), t(\theta_{N-k-1})) = (q^{FB}(\theta_{N-k-1}), t^{FB}(\theta_{N-k-1}))$$

Proper Robustness and Efficiency: Proof

Result 1: $M = (q, t) \in \mathscr{M}$ properly robust $\implies$ $V(q(\theta)) - t(\theta)$ non-increasing.

Proof of Result 1: Suppose contrary, i.e., $\exists j$ s.t.

$$V(q(\theta_{j-1})) - t(\theta_{j-1}) < V(q(\theta_j)) - t(\theta_j) \quad (*)$$

Let $J := \sup\{j : (*) \text{ holds}\}$. Consider $M^+ = (q^+, t^+)$ s.t.

$$\begin{cases} (q^+(\theta_j), t^+(\theta_j)) = (q(\theta_J), t(\theta_J)) & \text{for all } j = 1, ..., J \\ (q^+(\theta_j), t^+(\theta_j)) = (q(\theta_j), t(\theta_j)) & \text{for all } j = J+1, ..., N \end{cases}$$

Clearly, $M^+ \in \mathscr{M}$: because $(q(\theta_J), t(\theta_J)) \succeq_J (q(\theta_j), t(\theta_j))$ for all $j \geq J$,
$(q(\theta_J), t(\theta_J)) \succeq_m (q(\theta_j), t(\theta_j))$ for all $m \leq J$ all $j \geq J$, as $\mathrm{t} - \theta\mathrm{q}$ has SCP.

Proper Robustness and Efficiency: Proof

Proof of Result 1 (continued): $w_{\mathrm{F}}(M^{+}) >_L w_{\mathrm{F}}(M)$ for any full-support F strongly adversarial wrt M. To see this, let $k^* \equiv \min\{k : f_k(\theta_J) > 0\}$.

- For all $\theta \geq \theta_J$, $V(q^{+}(\theta)) - t^{+}(\theta) = V(q(\theta)) - t(\theta)$.
- For any $\theta < \theta_J$, if $f_k(\theta) > 0$ for some $k \leq k^*$, then $\theta \succcurlyeq_{\mathrm{F}} \theta_J$ and hence

  $$V(q(\theta)) - t(\theta) \leq V(q(\theta_J)) - t(\theta_J)$$

  because F strongly adversarial wrt M.
- Hence, for any $k = 1, ..., k^*$,

  $$\sum_{\theta \in \Theta} f_k(\theta) \cdot (V(q(\theta)) - t(\theta)) \leqslant \sum_{\theta \in \Theta} f_k(\theta) \cdot \left(V(q^{+}(\theta)) - t^{+}(\theta)\right)$$

- Furthermore,

  $$\sum_{\theta \in \Theta} f_{k^*}(\theta) \cdot (V(q(\theta)) - t(\theta)) < \sum_{\theta \in \Theta} f_{k^*}(\theta) \cdot \left(V(q^{+}(\theta)) - t^{+}(\theta)\right)$$

- Hence $w_{\mathrm{F}}(M^{+}) >_L w_{\mathrm{F}}(M)$. Therefore, M not properly robust.

Proper Robustness implies FB Efficiency: Proof

Result 2: $M = (q, t) \in \mathscr{M}$ properly robust $\implies$ M robust.

Proof of Result 2: Because M properly robust, $\exists$ full-support F strongly adversarial wrt M s.t. M F-optimal. That F strongly adversarial implies F adversarial. This means M robust (and hence WCO). Q.E.D.

- In turn, this implies $(q(\theta_N), t(\theta_N)) = (q^{FB}(\theta_N), t^{FB}(\theta_N))$

Proper Robustness and Efficiency: Proof

Result 3: $M = (q, t) \in \mathscr{M}$ properly robust and $(q(\theta_i), t(\theta_i)) = (q^{FB}(\theta_i), t^{FB}(\theta_i))$ for all $i = r, ..., N \implies$

$$(q(\theta_{r-1}), t(\theta_{r-1})) = (q^{FB}(\theta_{r-1}), t^{FB}(\theta_{r-1}))$$

Proof of Result 3: Suppose not. Then $w_{\mathrm{F}}(M^{FB}) >_L w_{\mathrm{F}}(M)$ for any full-support F strongly adversarial wrt M. To see this, first observe

$$V(q^{FB}(\theta_j)) - t^{FB}(\theta_j) > V(q(\theta_{r-1})) - t(\theta_{r-1}) \qquad \forall j < r \qquad (**)$$

In particular, for $j = r - 1$,

$$V(q^{FB}(\theta_{r-1})) - t^{FB}(\theta_{r-1}) > V(q(\theta_{r-1})) - t(\theta_{r-1})$$

This follows from

$$(q^{FB}(\theta_{r-1}), t^{FB}(\theta_{r-1})) = \arg\max_{\mathrm{q},\mathrm{t}} \{V(\mathrm{q}) - \theta_{r-1}\mathrm{q} - \Pi(\mathrm{q}, \mathrm{t}; \theta_{r-1})\}$$

$$\begin{aligned} \text{s.t. } \Pi(\mathrm{q}, \mathrm{t}; \theta_{r-1}) &\equiv \mathrm{t} - \theta_{r-1}\mathrm{q} \\ &\geq t^{FB}(\theta_r) - \theta_{r-1}q^{FB}(\theta_r) \end{aligned}$$

Because $V(q^{FB}(\theta)) - t^{FB}(\theta)$ non-increasing, $(**)$ follows.

Proper Robustness and Efficiency: Proof

Proof of Result 3 (continued):
Let $J = r - 1$ if $V(q(\theta_{r-2})) - t(\theta_{r-2}) > V(q(\theta_{r-1})) - t(\theta_{r-1})$. Else,

$$J \equiv \min\{j : V(q(\theta_j)) - t(\theta_j) = V(q(\theta_{r-1})) - t(\theta_{r-1})\}$$

Because M properly robust, $V(q(\theta)) - t(\theta)$ non-increasing. Hence, for any $\theta < \theta_J$ and any $j = J, ..., N$, $\theta_j >_{\mathrm{F}} \theta$ under any full-support F strongly adversarial wrt M. But then, $w_{\mathrm{F}}(M^{FB}) >_L w_{\mathrm{F}}(M)$. In fact, let $\hat{k} \equiv \min\{k : f_k(\theta_{r-1}) > 0\}$.

- For all $k < \hat{k}$, previous property and fact that F strongly adversarial wrt M,

$$\sum_{\theta \in \Theta} f_k(\theta) \cdot (V(q(\theta)) - t(\theta)) \leq \sum_{\theta \in \Theta} f_k(\theta) \cdot \left(V(q^{FB}(\theta)) - t^{FB}(\theta)\right)$$

- Furthermore, for $k = \hat{k}$,

$$\sum_{\theta \in \Theta} f_{\hat{k}}(\theta) \cdot (V(q(\theta)) - t(\theta)) < \sum_{\theta \in \Theta} f_{\hat{k}}(\theta) \cdot \left(V(q^{FB}(\theta)) - t^{FB}(\theta)\right)$$

- Hence, induction step applies. Q.E.D.

- Results 1-3 imply that if M **properly robust** then $M = M^{FB}$ (i.e., FB-efficient)

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