

NET 2021 Power Round

Wilson division, Microeconomics

April 2021

Instructions

This is the **microeconomics portion** of the Wilson division of the 2021 Northwestern Economics Tournament Power Round. There are three questions of *unequal* weight, accounting for a weighted *half* your score for the Power Round. You are encouraged to work together on these questions. Answer each question as clearly and succinctly as possible. You may write on a blank sheet of paper where you *clearly indicate* where your answer to each part is. If you are unsure of your answer, take your best guess: there is no penalty for incorrect answers. It is recommended you spend approximately an hour on this portion. Remember, we do *not* share your answers or scores with Northwestern admissions, nor do we keep them for ourselves. You are not expected to know how to answer each question on the exam; rather, this test is designed to assess your economic and formal reasoning skills. Have fun, and good luck!

Problem 1: Risk Aversion (12 Points)

Part A (1) Consider flipping a fair coin. Suppose the monetary payoff is \$1 if it lands heads and \$0 if it lands tails. This is an example of a *random variable*, which associates probabilities with random outcomes. The expected value of a discrete random variable is calculated by summing the numerical value times its corresponding probability. For example, the expected payoff of the above coin flip bet is $Pr(\text{heads}) * 1 + Pr(\text{tails}) * 0 = \0.5 . What's the expected payoff of 100 such coin flips? (Hint: the expected value of a sum of some set of random variables is the sum of the expected values of those random variables)

Part B (2) An *expected value maximizer* always tries to maximize the expected monetary payoff. We also call such a decision maker *risk neutral*. Suppose you are a value maximizer and you are faced with the following two options: you either pay \$45 to enter the previous game of 100 coin flips, or you pay \$10 for your lack of courage. Explain which option you should choose. (Hint: a mathematical argument is not necessary, but is more likely to crystallize your logic.)

Part C (4) Instead of maximizing expected value (i.e. maximizing expected wealth), suppose we had some type of individual who wants to achieve the highest *expected utility*, which is some abstract happiness value represented as a function of an individual's wealth. Suppose your wealth is \$12.30 right now, and you have the option to play a different game (the above game is not available anymore): we flip a coin once and you get \$6 if it lands heads and lose \$1 if it lands tails. You can either pay \$2.30 and play the game or you can choose to stay with your current wealth. If you are an expected value maximizer, what would you choose? If you are maximizing your expected utility function $u(w) = \sqrt{w}$, where w is your wealth at the end of the game, what would you choose? (Hint: When calculating expected utility, calculate the terminal utility for each outcome and sum them using their probabilities as weights)

Part D (*The St. Petersburg Paradox*) (3) Consider the bet described by the following procedure:

- Flip a coin: if it lands heads, you win \$1.
- If tails, flip it again; if now it lands heads, you win \$2.
- If tails, flip it once more; if now it lands heads, you win \$4.
- ...
- If it lands heads for the first time on the n^{th} toss, you win $\$2^{n-1}$
- The probability of the first heads occurring on the n^{th} toss is $\frac{1}{2}^n$

If you are an expected value maximizer, how much would you be willing to pay to enter this bet? If you are an expected utility maximizer, explain in words why you might not want to enter this bet for some amount of money.

Bonus (2) If your utility function is $\log_2(w)$ and your current wealth level is \$10, would you be willing to pay \$10 to enter this bet?

Problem 2: The Cournot Duopoly (18 Points)

Suppose two profit-maximizing firms operate in the same market and sell identical products. The products have a marginal cost of 2 per unit and there are zero fixed costs. The market price depends on the total quantity of goods in the market. The inverse demand function is

$$P = 102 - Q$$

where $Q = q_1 + q_2$. Thus, an equivalent expression for the inverse demand function is:

$$P = 102 - (q_1 + q_2)$$

This scenario is known as a duopoly. In a duopoly, both firms have market power, but the price depends on how much each firm produces. Thus, the firms are interdependent – any decision they make will depend on the decision the other firm makes.

Part A (2) Suppose both firms have equal access to information and decide what quantity to produce at the same time. Write down the profit equations for each firm in terms of q_1 and q_2 .

Part B (3) Because the price depends on the total quantity produced, marginal revenue for each firm also depends on the total quantity produced. For Firm 1, $MR_1 = 102 - 2q_1 - q_2$ and for Firm 2, $MR_2 = 102 - 2q_2 - q_1$. Calculate the quantity Firm 1 will produce in terms of q_2 and the quantity Firm 2 will produce in terms of q_1 .

Part C (3) How much will each firm produce? What will be the equilibrium price?

Part D (2) How much profit does each firm make?

Part E (3) The example above is known as a Cournot Duopoly. Suppose now that Firm 1 operates as a monopoly. The marginal revenue for Firm 1 is now $MR = 100 - 2Q$. How much will Firm 1 produce? What is the equilibrium price?

Part F (1) How much profit does Firm 1 make as a monopolist?

Part G (4) What happens to Firm 1 when Firm 2 enters the market? Who benefits and who loses as a result of the duopoly?

Problem 3: The Second Price Auction (18 points)

This problem addresses the first fundamental problem in the theory of auctions (for which Bob Wilson and Paul Milgrom won the 2020 Nobel Prize in economics.) Specifically, we will prove the incentive compatibility and Pareto efficiency of the second price auction, a special type of direct mechanism.

Consider the following. An impartial auctioneer is seeking to auction a single good for which some number n agents may submit bids. Each agent $1 \leq i \leq n$ has valuation $v_i \in [0, 1]$. The auction proceeds as follows: the auctioneer collects sealed bids from each of the participants, and ranks them from highest to lowest. The individual with the highest bid wins, but they pay the second highest bidder's bid. So if there are three bidders and the bids are $(8, 2, 0)$, the person with bid 8 wins and pays 2. We will assume that bids are drawn from a uniform distribution; this means that any value occurs with equal probability¹, so $Pr(v_j < \varepsilon) = \varepsilon$. Note in this problem, we denote b_i as the bid player i makes, v_i as their valuation, and p_i as the amount they bid if they win (so the bid of the next-highest bidder).

Part A (1) Fix some player i such that they bid b_i and would pay p_i if they win. What is their payoff function? (Note that there are two cases, with two different functions)

Part B (1) Now assume that there are two players, 1, 2. What are their payoffs if $b_1 < b_2$? If $b_1 > b_2$? (Note we may ignore $b_1 = b_2$, as it almost surely does not occur).

Part C (4) Since our payoffs are symmetric, we may without loss of generality show that it is dominant for player 1 to bid $b_1 = v_1$ regardless of the other player's strategy (that is, bidding one's true value is the *unique strictly dominant strategy*).

Part I Show that if $b_1 < v_1$, then there exists a value of b_2 such that bidding b_1 makes player 1 worse off; moreover, show that for all bids b_2 , bidding b_1 makes player 1 no better off than bidding v_1 .

Part II Show that if $b_1 > v_1$, then for all bids b_2 , bidding v_1 will give agent 1 with at least as much utility as bidding b_1 .

Part D (2) Explain intuitively and from the definition of Nash equilibrium why the bidding strategy $b_i = v_i$ (that is, both players bid their true values) is a Nash equilibrium (that is, if player 1 is playing $b_1 = v_1$, then player 2 cannot do better than playing $b_2 = v_2$).

Part E (3) Show the results from Parts (C) and (D) hold even when there are more than 2 players (note: you can just augment the argument you have already given).

Part F (2) Explain intuitively why the equilibrium in Part (D) and (E) of second price auction is *Pareto efficient*: that is, there does not exist a different allocation that can make everyone (weakly) better off.

Part G (2) Recall the auction is a sealed-bid auction; that is, each player i only knows the value of their bid, b_i , and none of the other bids. So far, we have considered that the auctioneer is fair and honest. This is, however, not always the case. Show that if we instead treat the auctioneer as a *player* in the game who (a) declares a winner, and (b), charges the winner some price p , then for any pair of first and second price bids (b_1, b_2) such that $b_1 < b_2$, then charging player 2 b_1 is *not* the revenue-maximizing price.²

Part H (3) Explain, using a similar argument as in Part (C), why if we were in a *first price auction* instead, (i.e. an auction where the highest bidder wins but pays the value of the highest bid) that it is in general *not* a Nash equilibrium for all players to bid their true valuations. Is the first price auction still incentive compatible? (That is, all players reveal their true valuations).

Bonus (2) Assume again that (v_1, v_2) are joint uniformly distributed over $[0, 1]$. Find the expected payment of player 1 in the second-price auction.

¹(That is, the probability that v_j is smaller than some valuation ε is exactly ε . This is a technical condition, and you will not need to worry too much about it).

²This is the *commitment problem* in auctions. Because of this, Google has recently switched from second price to first price auctions, which are still Pareto efficient but no longer incentive compatible. This problem is structural; see Li and Akbarpour, *Credible Auctions: A Trilemma*, Econometrica 2020.