NET 2023 Power Round

Introductory Division

March 2023

Instructions

This test consists of six questions. While you are free to attempt all six questions, we will only grade your four best-performing questions, regardless of how well you do on the other two. A question's point value is *not* informative of its difficulty; although questions have different point values, each question is weighted independently of its point value in your final cumulative score. After normalizing point-values of each question to the same weight, your cumulative score will be calculated as the sum of the scores of your four best-performing questions. You are encouraged to work together on these questions. Answer each question as clearly and succinctly as possible. You may write on a blank sheet of paper where you *clearly indicate* where your answer to each part is. If you are unsure of your answer, take your best guess: there is no penalty for incorrect answers. If you find yourself stuck on a question, skip it and return to it at the end if necessary. You will have two hours (120 minutes) to complete the exam. Remember, we do *not* share your answers or scores with Northwestern admissions, nor do we keep them for ourselves. You are not expected to know how to answer each question on the exam; rather, this test is designed to assess your economic and formal reasoning skills. Have fun, and good luck!

Problem 1: Tariffs (20 points)

Preliminaries Suppose the domestic market for pencils in NorthwesternLand is defined by the following demand and supply functions, respectively:

$$P(Q_d) = 10 - Q_d \tag{1}$$

$$P(Q_s) = Q_s \tag{2}$$

where P is price of pencils, Q_d is quantity demanded of pencils and Q_s is quantity supplied of pencils.

Part A *(1 point)* Calculate the equilibrium quantity and price for this domestic market.

Part B *(2 points total)* Now suppose that global producers enter the market. Assuming that global production quantities greatly outnumber domestic quantities, global supply can be approximated by $P = 2$. Assume consumers will buy from domestic producers first if prices are equivalent, and that they buy the lowest price.

(1) *(*1 point) What is the quantity of pencils domestic producers will sell?

(2) *(*1 point) What is the total quantity of pencils that will consumers will buy (from domestic and global producers)?

Part C *(2 points)* Based on the demand equation alone, what is the difference between the price consumers would be willing to pay in a hypothetical market where only one pencil is produced and the price they are required to pay in actual the market? What about in a market where two pencils are produced?

Part D *(3 points)* Consumer surplus (CS) is the net benefit consumers gain due to the difference between the price they are willing to pay for each unit and the price they are required to pay in the market. Assuming continuous units, what is the CS in the domestic pencil market of NorthwesternLand? *(Hint: Make a supply and demand graph. What area on the graph corresponds to consumer surplus?)*

Part E *(4 points)* Producer surplus (PS), the supplier analogue to CS, is the net benefit producers gain due to the difference in the price they are able to sell it for in the market and the price they are willing to sell for each unit. Assuming continuous units, what is the PS in the domestic pencil market of NorthwesternLand? *(Hint: Make a supply and demand graph. What area on the graph corresponds to consumer surplus?)*

Part F *(1 point)* Now suppose the government of NorthwesternLand imposes a tariff of $2 per unit pencil. What is the quantity of pencils that consumers will buy?

Part G *(4 points)* Calculate the CS and PS after this tariff. What is the change in each after the tariff?

Part H *(1 points)* Calculate how many dollars the government would collect from the tariff.

Part I *(2 points)* Sum the collected revenue and changes in surplus. Give an economic intuition as to what this sum means.

Problem 2: Many Macroeconomic Multipliers (25 points)

This problem starts by developing an intuitive approach to consumer spending. It then introduces the concept of marginal propensity to consume and provides insight into the mathematics behind the amplifying effects small changes in GDP can have on the overall macroeconomy.

Preliminaries We think about consumer spending as being divided between two behaviors: spending and saving.

Part A *(4 points total)* Interest Rates, Expected Salary Increases, Unemployment, Aggregate Price Level Changes (Inflation/Deflation), Changes in Tax Rates, Expectations about the economy

(1) *(2 points)* From the previous list of economic factors, pick two of them and explain how they might incentivize a consumer to **spend**.

(2) *(2 points)* Now pick two different factors and explain how they might incentivize a consumer to **save**.

We call the proportion of the marginal dollar a consumer spends the marginal propensity to consume (MPC). The total amount spent by an individual consumer (C) can be represented as a relation between their MPC, Y (their total income), T (the total amount of taxes paid), and A (autonomous consumption, the amount a consumer spends independently of their income).

Part B *(4 points)* Using economic intuition, determine the consumption function, a mathematical equation which expresses C in terms of Y, T, A, and MPC. Explain why the function has this form.

We now adopt a macroeconomic perspective on consumer spending. GDP is a measure of the market value of all goods and services produced and sold in a specific time period in countries. One simple way of calculating GDP is the following equation:

$$\text{GDP} = C+G+I+X$$

where C is total consumption (as determined by the consumption function), G is government expenditure, I is investment demand, and X is net exports.

When considering the economic effect of someone making an expenditure, economists predict that the total change in GDP will be much larger than the actual initial value of the expenditure. The rest of the equation explains the mathematics and intuition behind this prediction.

Part C *(4 points)* Suppose the government spends $100 million on goods. To whom does this money go to? What do they do with that money? Who gets the money then? What do they do with it?

It turns out this process iterates infinitely. If we assume everyone has the same MPC, we can express this using infinite sums. To determine the sum of an infinite geometric series with $|r|< 1$, denoted $a_0, a_1, ...$ use the following equation:

$$a_0/(1-r)$$

where a_0 is the initial term and r is the ratio between terms.

Part D *(3 points)* Return to the scenario in Part C. Suppose everyone has an MPC of 0.5. What would we expect the total change in GDP to be?

Part E *(10 points total)* It turns out changes in GDP follow a strict mathematical rule. Let ΔA, ΔY and ΔT denote the dollar amount of an initial change in autonomous spending, total income, and total taxes, respectively. The resulting net change on GDP is determined by the multiplier for each variable. Follow the steps to calculate the respective multipliers.

(1) *(2 point)* Using information from previous parts, express an equation which relates variables in the consumption function to GDP.

(2) *(4 points)* Suppose an initial increase in the amount of autonomous consumption of ΔA. Determine the resulting net change in GDP. Your answer should be in terms of ΔA and MPC. *(Hint: What is the initial increase in consumption (and therefore GDP)? Using the logic from Parts C and D, determine how the change in consumption would play out.)*

(3) *(4 points)* Suppose an initial increase in the amount of total income of ΔT. Determine the resulting net change in GDP. Your answer should be in terms of ΔT and MPC. *(Hint: use the same process outlined in Part b.)*

Problem 3: Economic Value Added (25 points)

In this problem we explore Economic Value Added, a financial model used to determine the worth of projects or investments. We will also engage with the cost of capital using a Weighted Average Cost of Capital model, subsidized by a Capital Asset Pricing model.

Preliminaries Suppose a firm is choosing between two projects. The first project, project A, costs \$10,000 and the other, project B, costs \$20,000. Both projects will bring returns two years later. The first project will bring in a total of \$12,000 and the second a total of \$24,500.

Part A *(1 point)* Divide the total profits for each project by the total cost, called the return on investment. Which project has a better return on investment?

Part B *(2 points)* Corporations are financed by equity and debt. Equity is typically bought as stock, while debt is typically given as loans. Why might debt and equity have different rates of return?

Part C *(2 points)* Suppose the firm has no liquid assets and chooses to source debt from a lender. The lender trusts this company with either, i) \$5,000 at 6% interest or ii) \$8,000 at 8% interest. The interest rate can be considered the *cost of debt*. The interest is compounded yearly. For each potential loan, calculated how much the firm will owe in two years.

Part D *(2 points)* To source the rest of the capital to begin one of these projects, the firm must sell equity in the form of stocks. The cost of equity is typically calculated as

$$Cost\ of\ Equity = Risk\text{-}Free\ Rate + \beta * Market\ Risk\ Premium \tag{3}$$

Where β, also known as volatility, is the correlation between the firm and the market, and the market risk premium is the difference in returns between investing in the market and a "Risk-Free" investment. Note that this is calculated yearly. What might be considered a "Risk-Free" investment, and what risks might it have?

Part E *(3 points)* Given a *Risk-Free Rate* of 4.5%, a β of 1.5, and a Market Risk Premium of 5%, calculate the *cost of equity*.

Part F *(3 points)* For each project, calculate the return required to make each investment break even after one year, for both loan options, given the previously calculated *cost of equity*.

Part G *(2 points)* Divide this number by the initial investments for each project and financing option. This is called the *Weighted Average Cost of Capital* (*WACC*).

$$WACC = \frac{Debt}{Debt + Equity} Cost\ of\ Debt + \frac{Equity}{Debt + Equity} Cost\ of\ Equity \tag{4}$$

Part H *(5 points)* Use the WACC to determine the return required for each project after two years.

Part I *(5 points)* Which project and which loan should be chosen? Why? Using Economic Value Added (EVA) may be helpful.

$$EVA = Return - Invest\ Capital * WACC \tag{5}$$

Problem 4: Bank Runs, Crashes, and Regulation (20 points)

In this model we consider two competing models of bank runs and the role of financial regulation, drawing on work from this year's Nobel Laureates.

Part A *(4 points)* Consider the following business model of a bank. 100 people each deposit \$100 each in your bank. You invest 98% of it in long-term loans (with interest), so that you can pay interest to the people who loaned you money and make a profit. You leave \$2 of it as cash on hand, in case people wish to withdraw cash. Suppose each person's withdrawal limit is \$1.

(1) *(1 point)* Suppose that, on average, 2% of individuals choose to withdraw cash every day. How much money should the bank expect to get withdrawn every day? Will they be able to service it with their on-hand investment?

(2) *(2 points)* Suppose the probabilities that people withdraw cash are independent, and each person has a 2% chance of withdrawing on a given day. What is the probability that more than two people will withdraw cash on the same day?

(3) *(1 point)* Given the answer in Part (2), do you think the bank's policy of keeping 2% of their deposits as cash-on-hand is reasonable? Why or why not?

Part B *(4 points)* Now suppose you know at least one person has withdrawn money earlier that day, and the bank has been unable to call in additional loans to handle withdrawal requests.

(1) *(1 point)* What is the probability, conditioned on one person already having withdrawn, that more than two people will withdraw on the same day?

(2) *(1 point)* Suppose that the (simple) interest rate[1] on your checking account *per year* is 6%. What is the daily interest rate?

(3) *(2 points)* Suppose that if more than two people withdraw their allowance, the bank must declare bankruptcy because it has no cash on hand, and you lose your entire deposit if they declare bankruptcy. If you want to maximize the expected value of your earnings, what should you do once one person has withdrawn?

Part C *(2 points)* Suppose everyone knows that at least one person has withdrawn from the bank. Using your answer from Part (B), what will they prefer to do?

Part D *(2 points)* Suppose now that everyone knows what will happen if at least one person withdraws from the bank. Will people prefer to withdraw their money if no one has withdrawn their cash or keep it in the bank for interest? Thus, what is the behavior you expect to see in this setting?

Part E *(3 points)* Suppose now that instead of a 2% chance of withdrawing on any given day, there are two possibilities:

- On a normal day, which occurs with probability 99%, there is a 0% chance of a withdrawal.
- On a "bad" day, which occurs with probability 1%, there is a 5% chance of withdrawal.

Based on your answer and reasoning in Parts (A)-(D), what do you expect to happen on a normal day? What about a bad day? Give one argument while this scenario may be more realistic then the model in Parts (A) through (D), and one reason why it may be less realistic.

Part F *(3 points)* Finally, assume that a government agency steps in and passes a law that says, if a bank is forced to declare bankruptcy, the government will reimburse all individuals who had deposited money the full value of their deposit.

[1] Assume the interest rate never compounds on the principal.

(1) *(1 point)* In both scenarios described above, what will individuals prefer to do?

(2) *(1 point)* Given your answer in Part (1), what is the expected payout of the government insurance agency to depositors over time?

(3) *(1 point)* Based on the model, give one argument in favor of the creation of deposit insurance, even though our model suggests banks are a relatively unstable institution.

Part G *(2 points)* Contrast the deposits in the US banking system with the recent instability of FTX and Terra-Luna cryptocurrencies, which are unregulated by the government. Which do you expect to be more stable over time? Explain.

Problem 5: Disappearing Debt (25 points)

In this problem we explore debt and deficits. In particular, we analyze what happens to debt over time in comparison to the gross domestic product and what the long term effect of debt is under differing conditions.

Part A *(1 point)* Suppose NETville spends \$100,000 this year but only receives \$50,000 in taxes. What is this government's deficit?

Part B *(2 points)* A country's debt is the sum of this year's deficit, the historical debt, and the interest on the historical debt. Suppose the historical debt of NETville is \$250,000, and they have a constant interest rate of 5%. What is NETville's national debt this year? (Do not forget to consider the deficit!)

Part C *(1 point)* NETville is known for its strong economy that consistently grows 10% per year. If the GDP of NETville was \$400,000, what is it this year?

Part D *(4 points)* What is the debt to GDP ratio of NETville this year? What was it last year?

Part E *(4 points)* Assume the government of NETville decides to enact policy next year to ensure there is no deficit and the interest rate and GDP growth rate stays the same. Calculate next year's debt, GDP, and debt-to-GDP ratio.

Part F *(3 points)* If the government of NETville maintains that policy and takes on no more deficit, what happens to the debt-to-GDP ratio of NETville?

Part G *(6 points)* Imagine the interest rate on NETville's debt is r, and the growth rate of NETville's GDP is g.

(1) *(2 points)* What would you expect to happen to the debt-to-GDP ratio of NETville when $g > r$?

(2) *(2 points)* What if $r = g$?

(3) *(2 points)* What if $g < r$?

Part H *(4 points)* Suppose NETville takes on an additional \$50,000 deficit every year at a 5% interest rate and its economy continues to grow at 10% every year. What do you expect to happen to the debt-to-GDP ratio?

Problem 6: Unraveling (20 points)

Preliminaries Suppose there are five students in your class. After an exam, the professor gives each student their test back, and each student knows their own grade. Now, the professor writes the following test score distribution on the blackboard: 90, 80, 70, 60, 50. The professor offers them a decision:

(1) accept their individual score on the test by turning their test in.

(2) accept the mean score of those who have not turned their tests in.

Part A *(1 point)* Calculate the mean score of the class.

Part B *(2 points)* Assume all students are only concerned with maximizing test scores, there is no cooperation, and all students act rationally. What should the student who received a score of 90 do? Explain.

Part C *(3 points)* Given the action of the student who received a 90, state what the student who received an 80 should do and justify the result mathematically.

Part D *(2 points)* Explain what will occur with the next two students.

Part E *(3 points)* If there is only one student who has not turned in their test, explain why they are indifferent between the two decisions.

Part F *(5 points)* Assume that students who are indifferent always choose to turn their test in. Does the pattern observed in the previous parts always occur for any set of test scores? Explain.

Part G *(4 points)* Now assume students do not know the other student's scores, but do know their own scores. Each student has a (potentially different) *belief* over the distribution of other's scores, and turns in their test if they are indifferent between keeping it and turning it in. Will this same pattern still occur? Explain.