NET 2023 Power Round

Introductory Division: Solutions

April 27, 2023

Instructions

This test consists of six questions. While you are free to attempt all six questions, we will only grade your four best-performing questions, regardless of how well you do on the other two. A question's point value is *not* informative of its difficulty; although questions have different point values, each question is weighted independently of its point value in your final cumulative score. After normalizing point-values of each question to the same weight, your cumulative score will be calculated as the sum of the scores of your four best-performing questions. You are encouraged to work together on these questions. Answer each question as clearly and succinctly as possible. You may write on a blank sheet of paper where you *clearly indicate* where your answer to each part is. If you are unsure of your answer, take your best guess: there is no penalty for incorrect answers. If you find yourself stuck on a question, skip it and return to it at the end if necessary. You will have two hours (120 minutes) to complete the exam. Remember, we do *not* share your answers or scores with Northwestern admissions, nor do we keep them for ourselves. You are not expected to know how to answer each question on the exam; rather, this test is designed to assess your economic and formal reasoning skills. Have fun, and good luck!

Problem 1: Tariffs (20 points)

Preliminaries Suppose the domestic market for pencils in NorthwesternLand is defined by the following demand and supply functions, respectively:

$$P(Q_d) = 10 - Q_d \tag{1}$$

$$P(Q_s) = Q_s \tag{2}$$

where P is price of pencils, Q_d is quantity demanded of pencils and Q_s is quantity supplied of pencils.

Part A *(1 point)* Calculate the equilibrium quantity and price for this domestic market. We solve for the intersection of demand and supply curve, yielding equilibrium quantity of 5, equilibrium price of 5. *One point for **both** the correct equilibrium price and quantity*

Part B *(2 points total)* Now suppose that global producers enter the market. Assuming that global production quantities greatly outnumber domestic quantities, global supply can be approximated by $P = 2$. Assume consumers will buy from domestic producers first if prices are equivalent, and that they buy the lowest price.

(1) *(*1 point) What is the quantity of pencils domestic producers will sell? We know pencils can be bought for \$2 from global producers. Therefore, consumers will pay no more than \$2 for each pencil. We solve for the intersection between P=2 and the domestic supply curve to find the quantity of pencils domestic producers would be willing to supply at P=2, yielding quantity of 2. *One point for the correct quantity*

(2) *(*1 point) What is the total quantity of pencils that will consumers will buy (from domestic and global producers)? We solve for the intersection of demand curve and global supply curve, yielding total quantity of 8. *One point for the correct total quantity*

Part C *(2 points)* Based on the demand equation alone, what is the difference between the price consumers would be willing to pay in a hypothetical market where only one pencil is produced and the price they are required to pay in actual the market? What about in a market where two pencils are produced? At Q=1, demand curve yields P=9 (the price consumers are willing to pay). At Q=2, demand curve yields P=8. In both markets, the price they are required to pay is \$2. Price difference when one pencil is produced is \$7, difference when two pencils are produced is \$6. *One point for each correct price difference* (Beginning from this question, some interpreted the question as referring to domestic suppliers rather than global suppliers. Full points were awarded for this alternative interpretation and answers will be denoted in red.) At Q=1, price difference is 4. At Q=2, price difference is 3.

Part D *(3 points)* Consumer surplus (CS) is the net benefit consumers gain due to the difference between the price they are willing to pay for each unit and the price they are required to pay in the market. Assuming continuous units, what is the CS in the domestic pencil market of NorthwesternLand? *(Hint: Make a supply and demand graph. What area on the graph corresponds to consumer surplus?)* CS is given by the area between the demand curve and P=2. Area of this triangle is (1/2)(8)(8)=32. *Three points for calculating area; two points if only identified the area on graph* Area is 25/2.

Part E *(4 points)* Producer surplus (PS), the supplier analogue to CS, is the net benefit producers gain due to the difference in the price they are able to sell it for in the market and the price they are willing to sell for each unit. Assuming continuous units, what is the PS in the domestic pencil market of NorthwesternLand? *(Hint: Make a supply and demand graph. What area on the graph corresponds to producer surplus?)* PS is given by the area between the domestic supply curve and P=2. Area of this triangle is (1/2)(2)(2)=2. *Four points for calculating area; three points if only identified the area on graph* Area is 25/2.

Part F *(1 point)* Now suppose the government of NorthwesternLand imposes a tariff of \$2 per unit pencil. What is the quantity of pencils that consumers will buy? Now the price of pencils from the consumers' perspectives is effectively \$2 + \$2 = \$4. At P=4, demand curve yields Q=6. *One point for the correct quantity*

Part G *(4 points)* Calculate the CS and PS after this tariff. What is the change in each after the tariff? New CS is given by the area between the demand curve and P=4. Area of this triangle is (1/2)(6)(6)=18. (New CS)-(Old CS)=18-32=-14. New PS is given by the area between the domestic supply curve and P=4. Area of this triangle is (1/2)(4)(4)=8. (New PS)-(Old PS)=8-2=6. *One point for new CS, one point for new PS, one point for change in CS, one point for change in PS* Change in CS is 11/2. Change in PS is -9/2.

Part H *(1 points)* Calculate how many dollars the government would collect from the tariff. International producers would provide 2 of the 6 pencils, government gets \$2 per pencil. (2)(2)=\$4 *One point for numerical amount government collects from tariff*

Part I *(2 points)* Sum the collected revenue and changes in surplus. Give an economic intuition as to what this sum means. -14+6+4=-4; this sum is the amount of utility that has been lost in Northwesternland as a result of the tariff. *One point for sum, one point for correct intuition (not necessary to provide "deadweight loss"* 11/2-9/2+4=5

Problem 2: Many Macroeconomic Multipliers (25 points)

This problem starts by developing an intuitive approach to consumer spending. It then introduces the concept of marginal propensity to consume and provides insight into the mathematics behind the amplifying effects small changes in GDP can have on the overall macroeconomy.

Preliminaries We think about consumer spending as being divided between two behaviors: spending and saving.

Part A *(4 points total)* Interest Rates, Expected Salary Increases, Unemployment, Aggregate Price Level Changes (Inflation/Deflation), Changes in Tax Rates, Expectations about the economy

(1) *(2 points)* From the previous list of economic factors, pick two of them and explain how they might incentivize a consumer to **spend**.
Possible answers included becoming employed, expected salary increases, decreases in tax rates, positive expectations about the economy, lower interest rates and inflation.

1 point is awarded for each correctly identified factor and accompanying explanation.

(2) *(2 points)* Now pick two different factors and explain how they might incentivize a consumer to **save**.
Possible answers included becoming unemployed, expected salary decreases, increases in tax rates, negative expectations about the economy, higher interest rates and deflation.

1 point is awarded for each correctly identified factor and accompanying explanation.

We call the proportion of the marginal dollar a consumer spends the marginal propensity to consume (MPC). The total amount spent by an individual consumer (C) can be represented as a relation between their MPC, Y (their total income), T (the total amount of taxes paid), and A (autonomous consumption, the amount a consumer spends independently of their income).

Part B *(4 points)* Using economic intuition, determine the consumption function, a mathematical equation which expresses C in terms of Y, T, A, and MPC. Explain why the function has this form.

$$C = A + MPC(Y - T) \tag{3}$$

Here's a possible explanation: The consumption function is a linear relationship between disposable income and consumer spending. Since autonomous consumption happens independently of disposable income, it can be thought of as the amount of consumer spending even when they have no disposable income. Therefore, it is the "y-intercept" of the consumption function. MPC tells us how much of each additional dollar of disposable income is spent, or the slope of the function between disposable income and consumption. Finally, disposable income is calculated as the difference of your income and how much of it you pay as taxes.

1 point was given for a correct function. 1 point was given for a correct discussion of autonomous consumption, disposable income and the MPC, respectively.

We now adopt a macroeconomic perspective on consumer spending. GDP is a measure of the market value of all goods and services produced and sold in a specific time period in countries. One simple way of calculating GDP is the following equation:

$$\text{GDP} = C+G+I+X$$

where C is total consumption (as determined by the consumption function), G is government expenditure, I is investment demand, and X is net exports.

When considering the economic effect of someone making an expenditure, economists predict that the total change in GDP will be much larger than the actual initial value of the expenditure. The rest of the problem explores the mathematics and intuition behind this prediction.

Part C *(4 points)* Suppose the government spends \$100 million on goods. To whom does this money go to? What do they do with that money? Who gets the money then? What do they do with it?
This money goes to consumers who then spend a portion of that money based on their MPC. Then, another set of consumers receives the money spent by the first group of benficiaries and they spend a portion of that money based on their MPC.

1 point is awarded for identifying that consumers are the initial beneficiaries. 1 point is awarded for identifying that they will spend that money. 2 points were awarded for identifying that the process repeats with the second group of consumers.

It turns out this process iterates infinitely. If we assume everyone has the same MPC, we can express this using infinite sums. To determine the sum of an infinite geometric series with $|r| < 1$, denoted $a_0, a_1, ...$ use the following equation:

$$a_0/(1-r)$$

where a_0 is the initial term and r is the ratio between terms.

Part D *(3 points)* Return to the scenario in Part C. Suppose everyone has an MPC of 0.5. What would we expect the total change in GDP to be?
We use the given formula and plug-in the proper values: $a_0 = \$100,000,000, r = 0.5$. This yields a correct final answer: $\$200,000,000$

1 point is awarded for a correct final answer. 1 point is awarded for correct substitution for a_0, and 1 point is awarded for correct substitution for r.

Part E *(10 points total)* It turns out changes in GDP follow a strict mathematical rule. Let ΔA, ΔY and ΔT denote the dollar amount of an initial change in autonomous spending, total income, and total taxes, respectively. The resulting net change on GDP is determined by the multiplier for each variable. Follow the steps to calculate the respective multipliers.

(1) *(2 point)* Using information from previous parts, express an equation which relates variables in the consumption function to GDP.

$$GDP = C + I + G + X = A + MPC(Y - T) + I + G + X \tag{4}$$

1 point is awarded for a correct substitution of the consumption function given in part (B) for C. 1 point is awarded for correct final answer.

(2) *(4 points)* Suppose an initial increase in the amount of autonomous consumption of ΔA. Determine the resulting net change in GDP. Your answer should be in terms of ΔA and MPC. *(Hint: What is the initial increase in consumption (and therefore GDP)? Using the logic from Parts C and D, determine how the change in consumption would play out.)* The change in autonomous spending is subject to the multiplier effect we observed in Parts C and D. Therefore, for an initial change of ΔA, we should expect a total change in GDP

of $\frac{\Delta A}{1-MPC}$.

1 point is awarded for recognizing that the change in GDP has a direct relationship with ΔA. 1 point is awarded for identifying that change in GDP has a direct relationship with MPC. 2 points were awarded for a final correct answer. Full points were awarded to correct answers.

(3) *(4 points)* Suppose an initial increase in the amount of total income of ΔT. Determine the resulting net change in GDP. Your answer should be in terms of ΔT and MPC. *(Hint: use the same process outlined in Part b.)* The change in taxation affects disposable income, and therefore the total initial change in consumption is given by $-MPC\Delta T$. This initial change is subject to the multiplier effect we observed in Parts C and D. Therefore, for an initial change of $-MPC\Delta T$, we should expect a total change in GDP of $\frac{-MPC\Delta T}{1-MPC}$.

1 point is awarded for recognizing that the change in GDP has an inverse relationship with ΔT. 1 point is awarded for recognizing that the initial change in consumption is given by $-MPC\Delta T$. 2 points were awarded for a final correct answer. Full points were awarded to correct answers.

Problem 3: Economic Value Added (25 points)

In this problem we explore Economic Value Added, a financial model used to determine the worth of projects or investments. We will also engage with the cost of capital using a Weighted Average Cost of Capital model, subsidized by a Capital Asset Pricing model.

Suppose a firm is choosing between two projects. The first project, project A, costs $10,000 and the other, project B, costs $20,000. Both projects will bring returns two years later. The first project will bring in a total of $12,000 and the second a total of $24,500.

Part A (1) Divide the total profits for each project by the total cost, called the return on investment. Which project has a better return on investment? Project B has a better return on investment. 1 point for the correct answer.

Part B (2) Corporations are financed by equity and debt. Equity is typically bought as stock, while debt is typically given as loans. Why might debt and equity have different rates of returns? Debt is to be paid back at specified intervals making it a less risky investment than equity, which is subject to market changes. 1 point for mentions of risk. 1 point for additional discussion.

Part C (2) Suppose the firm has no liquid assets and chooses to source debt from a lender. The lender trusts this company with either, i) $5,000 at 6% interest or ii) $8,000 at 8% interest. The interest rate can be considered the *cost of debt*. The interest is compounded yearly. For each potential loan, calculated how much the firm will owe in two years. i) 5300, 5618 ii) 8640, 9331.2. 1 point for each correct answer. 1 point partial credit if a student gets both correct for the first year.

Part D (2) To source the rest of the capital to begin one of these projects, the firm must sell equity in the form of stocks. The cost of equity is typically calculated as

$$\textit{Cost of Equity} = \textit{Risk-Free Rate} + \beta * \textit{Market Risk Premium}$$

Where β, also known as volatility, is the correlation between the firm and the market, and the market risk premium is the difference in returns between investing in the market and a "Risk-Free" investment. Note that this is calculated yearly. What might be considered a "Risk-Free" investment, and what risks might it have? Normally treasury bonds or government bonds or any other kind of government investment is considered risk-free since the government is unlikely to fail. Some risks could be the government failing or failing to pay back parts of its debt. Additionally these bonds are not liquid and have significant inflation risk. 1 point for mentioning a risk-free investment and 1 point for a discussed risk.

Part E (3) Given a *Risk-Free Rate* of 4.5%, a β of 1.5, and a Market Risk Premium of 5%, calculate the *cost of equity*. The correct answer is 12. The formula is stated above. 3 points for a correct answer, possible 1 point partial credit for messing up one of the pieces.

Part F (3) For each project, calculate the return required to make each investment break even after one year, for both loan options, given the previously calculated *cost of equity*. From here on out there will be four options for each problem. The projects are denoted A and B, the loan options are i and ii. The formula for this question is

$$Requiredyield = investment - loanamount * 1.12 + loanamount * interestrate.$$

Ai) 10900, Aii) 10880, Bi) 22100, Bii) 22080. .5 points for correct answer, 1 point for correct formulas.

Part G (2) Divide this number by the initial investments for each project and financing option. This is called the *Weighted Average Cost of Capital* (*WACC*).

$$WACC = \frac{Debt}{Debt + Equity} Cost\ of\ Debt + \frac{Equity}{Debt + Equity} Cost\ of\ Equity$$

This problem could run into problems with the previous question, so treat the answer as correct. Ai) 1.09 Aii) 1.088 Bi) 1.105 Bii) 1.104. .5 points per correct answer, partial credit available up to 1 point for correct formula. Some students may have extrapolated the formula across two years and solved for the WACC which is also treated as correct.

Part H (5) Use the WACC to determine the return required for each project after two years.

In this problem they should square the WACC and multiply it by the investment. Ai) 11881 Aii)11837 Bi) 24420 Bii) 24376. Students might note this is different than the rates than if the rates were calculated independently. That is because we used a 1 year WACC. Answers that used a 2 year WACC were also considered correct. 1 point per correct answer with 1 point given to students who use the correct formula.

Part I (5) Which project and which loan should be chosen? Why? Using Economic Value Added (EVA) may be helpful.

$$EVA = Return - Invest\ Capital * WACC$$

Answers can consider profits or rates of return but need to be explained. They will receive 2 points for showing the EVAs and 2 points for making an argument for their findings. The students will also receive 1 point for discussing the model

Problem 4: Bank Runs, Crashes, and Regulation (20 points)

In this model we consider two competing models of bank runs and the role of financial regulation, drawing on work from this year's Nobel Laureates.

Part A *(4 points)* Consider the following business model of a bank. 100 people each deposit \$100 each in your bank. You invest 98% of it in long-term loans (with interest), so that you can pay interest to the people who loaned you money and make a profit. You leave \$2 of it as cash on hand, in case people wish to withdraw cash. Suppose each person's withdrawal limit is \$1.

(1) *(1 point)* Suppose that, on average, 2% of individuals choose to withdraw cash every day. How much money should the bank expect to get withdrawn every day? Will they be able to service it with their on-hand investment?
The bank expects \$2 to get withdrawn every day. Because they be able to service it. Yes, they can. *One point was given for noting \$2 would get withdrawn.*

(2) *(2 points)* Suppose the probabilities that people withdraw cash are independent, and each person has a 2% chance of withdrawing on a given day. What is the probability that more than two people will withdraw cash on the same day?
There is a total of a $\frac{2^3}{100^3} = \frac{1}{125000}$ chance that more than two people withdraw cash, so it is reasonable. *One point was given for the numerical answer, and a second for the justification.*

(3) *(1 point)* Given the answer in Part (2), do you think the bank's policy of keeping 2% of their deposits as cash-on-hand is reasonable? Why or why not?
Yes, it is reasonable, since the probability of going over the withdrawal limit is arbitrarily small. *One point was given for any reasonable justification of any position.*

Part B *(4 points)* Now suppose you know at least one person has withdrawn money earlier that day, and the bank has been unable to call in additional loans to handle withdrawal requests.

(1) *(1 point)* What is the probability, conditioned on one person already having withdrawn, that more than two people will withdraw on the same day?
The probability is $\frac{2^2}{100^2}$ since we know at least one person has already withdrawn; this is $\frac{1}{2500}$. *One point was given for the correct distribution.*

(2) *(1 point)* Suppose that the (simple) interest rate[1] on your checking account *per year* is 6%. What is the daily interest rate?
The daily interest rate is $\frac{0.06}{365} \approx 0.000164$ or around 0.0164%. *One point was given for the correct computation.*

(3) *(2 points)* Suppose that if more than two people withdraw their allowance, the bank must declare bankruptcy because it has no cash on hand, and you lose your entire deposit if they declare bankruptcy. If you want to maximize the expected value of your earnings, what should you do once one person has withdrawn?
Note that

$$(0.000164)(100)(0.9996) + (0.0004)(-100) < 0$$

is the expected value of staying in the bank for a day, conditional on one person having withdrawn. As a result, you would withdraw. *One point was given for recognizing the expected value of continuing in the bank was negative; a second point was given for a correct conclusion that you should withdraw from the bank.*

[1] Assume the interest rate never compounds on the principal.

Part C *(2 points)* Suppose everyone knows that at least one person has withdrawn from the bank. Using your answer from Part (B), what will they prefer to do?
If everyone knows at least one person has withdrawn, everyone else will withdraw. *Both points were given for concluding that "bank run" behavior would arise.*

Part D *(2 points)* Suppose now that everyone knows what will happen if at least one person withdraws from the bank. Will people prefer to withdraw their money if no one has withdrawn their cash or keep it in the bank for interest? Thus, what is the behavior you expect to see in this setting?
Because they know as soon as one person withdraws, a bank run occurs, the probability of a bank run is now the probability that one person randomly withdraws from the bank. But this is 2%, which is larger than the probability above. As a result, the expected value of the bank is negative, and everyone will withdraw (or the bank will not exist in the first place). *One point was given for noting the expected value of keeping money in the bank is negative even before anyone has withdrawn. A second point was given for a relevant conclusion.*

Part E *(3 points)* Suppose now that instead of a 2% chance of withdrawing on any given day, there are two possibilities:

– On a normal day, which occurs with probability 99%, there is a 0% chance of a withdrawal.

– On a "bad" day, which occurs with probability 1%, there is a 5% chance of withdrawal.

Based on your answer and reasoning in Parts (A)-(D), what do you expect to happen on a normal day? What about a bad day? Give one argument while this scenario may be more realistic then the model in Parts (A) through (D), and one reason why it may be less realistic.
On a normal day, no one withdraws, so the bank will exist because there is zero probability of a run, and people will want to earn interest. On a bad day, a run will immediately occur. This is more realistic, since it

(1) Gives a reason why a bank might exist (good days are frequent and happen with enough probability the expected withdrawal probability is small enough to create positive expected value to storing in the bank).

(2) Explains why we might see sudden "shocks" in a bank run after a long period of instability, mirroring what we see in the real world.

One point was given for noting that the expected value of storing in the bank (over both good and bad days) was now positive. A second point was given for noting this implies a bank would likely exist. A third point was given for a reasonable justification for why this was more realistic.

Part F *(3 points)* Finally, assume that a government agency steps in and passes a law that says, if a bank is forced to declare bankruptcy, the government will reimburse all individuals who had deposited money the full value of their deposit.

(1) *(1 point)* In both scenarios described above, what will individuals prefer to do?
In the scenarios of Part (D) and (E), with government insurance, people will want to save in the bank. *One point was given for the correct conclusion.*

(2) *(1 point)* Given your answer in Part (1), what is the expected payout of the government insurance agency to depositors over time?
The expected payout of government insurance to depositors over time will be zero, since a bank run will never occur given that they will never lose anything if the bank hits its withdrawal limit and runs out of cash on hand *One point was given for any justification for why the expected payout is zero.*

(3) *(1 point)* Based on the model, give one argument in favor of the creation of deposit insurance, even though our model suggests banks are a relatively unstable institution.
The model suggests deposit insurance is a costless way to prevent bank runs and establish a functioning banking system. *One point was given for any reasonable justification.*

Part G *(2 points)* Contrast the deposits in the US banking system with the recent instability of FTX and Terra-Luna cryptocurrencies, which are unregulated by the government. Which do you expect to be more stable over time? Explain.
Because cryptocurrencies have no FDIC insurance, we expect that, based on the model,

(1) Cryptocurrencies will likely face longer peirods of instability.

(2) Those periods of instability are likely to be more pronounced.

(3) After a specific cryptocurrency faces a crash, it is unlikely faith will be restored to that currency (unlike bank runs and US currency).

(4) . . . and other possibilities, that are not features of banks.

Ont point was given for each relevant contrast and discussion.

Problem 5: Disappearing Debt (25 points)

In this problem we explore debt and deficits. Particularly what happens to debt over time in comparison to the gross domestic product and what the long term effect of debt is under differing conditions.

Part A (1) Suppose NETville spends $100,000 this year, but only receives $50,000 in taxes. What is this government's deficit? $50,000. 1 point for the correct answer.

Part B (2) A country's debt is the sum of this year's deficit, the historical debt, and the interest on the historical debt. Suppose the historical debt of NETville is $250,000, and they have a constant interest rate of 5%. What is NETville's national debt this year? (Don't forget to consider the deficit!) $312,500. 2 points for the correct answer. If the student didn't realize the previous question was maintained and put 262,500, 1 point.

Part C (1) NETville is known for its strong economy that consistently grows 10% per year. If the GDP of NETville was $400,000, what is it this year? $440,000. 1 point for the correct answer.

Part D (4) Was the debt to GDP ratio of NETville this year? What was it last year? 2 points for for any rounded amount of 312,500/440,000 f0r this year, 0.7102 and 2 points for 250,000/400,000 last year.

Part E (4) Assuming the government of NETville decides to enact policy next year to ensure there is no deficit and the interest rate and GDP growth rate stays the same. Calculate next year's debt, GDP, and debt-to-GDP ratio. 1 point for debt: $328,125. 1 point for GDP: $484,000. 1 point for correct formula of debt-to-GDP and 1 point for correct answer: .6779 or any rounded example.

Part F (3) If the government of NETville maintains that policy and takes on no more deficit, what happens to the debt-to-GDP ratio of NETville? Full points for saying it will decrease, partial credit available for illustrating that the different rates of growth will cause changes.

Part G (6) Imagine the interest rate on NETville's debt is r, and the growth rate of NETville's GDP is g. What do you expect to happen to the debt-to-GDP ratio of NETville when i) g > r ii) r = g iii) g < r? 2 points for each part. for i, correct answer is debt-to-GDP will decrease, for part ii. correct answer is debt-to-GDP will stay the same , for part iii. correct answer is debt-to-GDP will increase.

Part H (4) Suppose NETville takes on an additional $50,000 deficit every year at a 5% interest rate and it's economy continues to grow at 10% every year. Using the equations for debt and debt-to-GDP what do expect to happen to the debt-to-GDP ratio? In the short term we expect the debt-to-GDP ratio to rise, but eventually the GDP will out grow the debt and it will begin to decrease. It is not necessary to solve for the specific periods. A bit more open ended but 1.5 point for short term discussion, 1.5 point for a long term discussion and 1 point for demonstrating a formula or examples.

$$GDP = 250000 * 1.05^x + 50000 * x$$

$$Debt = 400000 * 1.10^x$$

Problem 6: Unraveling (20 points)

Preliminaries Suppose there are five students in your class. After an exam, the professor gives each student their test back, and each student knows their own grade. Now, the professor writes the following test score distribution on the blackboard: 90, 80, 70, 60, 50. The professor offers them a decision:

(1) accept their individual score on the test by turning their test in.

(2) accept the mean score of those who have not turned their tests in.

Part A *(1 point)* Calculate the mean score of the class. $(90+80+70+60+50)/5 = 70$ *One point for mean score*

Part B *(2 points)* Assume all students are only concerned with maximizing test scores, there is no cooperation, and all students act rationally. What should the student who received a score of 90 do? Explain. The student who received a score of 90 would receive 90 if they turned their test in, and 70 if they did not. Since 90>70, they would prefer to turn their test in. *One point for identifying that the student should turn it in, one point for valid explanation*

Part C *(3 points)* Given the action of the student who received a 90, state what the student who received an 80 should do and justify the result mathematically. After the student who received a 90 turns their test in, the mean score is now $(80+70+60+50)/4 = 65$. Since 80>65, the student who received an 80 should turn their test in. *One point for calculating the new mean score, one point for comparing 80 to the new mean score, one point for identifying that the student should turn it in*

Part D *(2 points)* Explain what will occur with the next two students. Following the same pattern, the next student's score will be the top score, and will therefore be greater than the mean score, so they will turn their test in. After they turn it in, the student after will have the top score, which will be greater than the mean score, so they will turn it in too. *Two points for the correct explanation*

Part E *(3 points)* If there is only one student who has not turned in their test, explain why they are indifferent between the two decisions. If there is only one student, the mean score is identical to their own score, so they would be equally well off whether they turned their test in or not. *Three points for the correct explanation*

Part F *(5 points)* Assume that students who are indifferent always choose to turn their test in. Does the pattern observed in the previous parts always occur for any set of test scores? Explain. For any set of test scores, the top score(s) will always be greater than or equal to the mean score. Therefore, if they turn their test when they are indifferent, each top scoring student will always turn their test in, and the same pattern emerges. *One point for identifying the pattern always occurs for any set of test scores, four points for the correct explanation*

Part G *(4 points)* Now assume students do not know the other student's scores, but do know their own scores. Each student has a (potentially different) *belief* over the distribution of other's scores, and turns in their test if they are indifferent between keeping it and turning it in. Will this same pattern still occur? Explain. For any distribution of scores, the students will be better off by turning in their tests. Therefore, regardless of what the students believe the score distribution to be, the pattern will always occur. *One point for identifying the pattern always occurs for any set of test scores, three points for the correct explanation*